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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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SEPTEMBER 1923

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THE
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VOLUME LVIII SEPTEMBER 1923 NUMBER 2

WATER-CELL TRANSMISSIONS AND PLANETARY TEMPERATURES

By DONALD H. MENZEL

ABSTRACT

Determination of planetary temperatures from water-cell transmission.—(1) *Theory.* The reflected solar energy depends upon the planet's size, distance, albedo, and phase-angle; the emitted energy depends also upon the effective temperature and the emissivity of the surface. The observed ratio of the two is greatly modified by absorption in the earth's atmosphere and the receiving instrument. The equation connecting the effective temperature with the water-cell transmission is derived. When the whole disk of the planet has been observed the only uncertain factor is its emissivity; when but part of the disk has been observed a further correction (now very uncertain, but capable of accurate measurement) must be applied. (2) *Results of radiometric observations* made by Coblentz at the Lick and Lowell observatories in 1914 and 1921–1922, when reduced by this method, show good agreement and indicate effective noon-day temperatures of 50° C. for *Venus*, −16° C. for *Mars*, and 120° C. for the *moon* (assuming high emissivity in all cases). These are in good agreement with the temperatures which would be maintained by solar radiation on a surface of small heat capacity. For *Jupiter and Saturn* the computed temperatures (−110° C. in both cases) are too high to be maintained by solar radiation, and must be attributed to the internal heat of the planets. These numerical values should be regarded as provisional.

Atmospheric transmission of long-wave radiation as a function of humidity.—The earth's atmosphere (owing mainly to its content of carbon dioxide and water-vapor) acts like a color-screen for long waves, transmitting most of the radiation between 9 μ and 12 μ and very little else. The net transmission of black-body radiation, for temperatures between 400° and 150° K., varies roughly as the cube of the temperature. For the range of humidity likely to be met in practice (0.5 to 4 cm precipitable water), the transmission can be represented by the product of two factors, one depending upon the air-mass and the other upon the total amount of water-vapor along the path. *Tables* of these factors are given for different temperatures of the source.

The accuracy of recent measures of planetary radiation encourages an examination of the relations of the observed data to the

temperatures of the planets. This has been undertaken at the suggestion of Professor Russell, who has prepared the outline of the theory given in section 1. The calculations which follow have been made by the writer.

1. *General theory.*—Much has been written on this subject, and the present summary has small claims to originality. The energy which we receive from a planet is composed partly of reflected solar radiation of short wave-length and partly of long-wave radiation from the planet's warm surface. The former is largely transmitted by a water cell and the latter stopped completely.

Let S be the amount of the former and P that of the latter, and let t' and t be the fractions of these amounts which are transmitted through the earth's atmosphere and the receiving instruments employed. The radiation observed without the water cell will then be $tP+t'S$, while with it, it will be $0.695\ t'S$—the numerical factor being the mean value determined by Coblentz.[1] If W is the "water-cell transmission," we then have

$$W=0.695\ t'S/(tP+t'S)$$

Or

$$\frac{tP}{t'S}=\frac{0.695}{W}-1. \tag{1}$$

In general, the portion of the planet's surface whose energy falls on the receiving thermopile will not be of uniform temperature. We may substitute a hypothetical "gray" surface of equal size and of temperature T and emissivity ϵ such that the radiation from this surface is equal to the integrated radiation from the observed region in amount, and as similar as possible in quality, and write

$$P=ka\epsilon T^4 \tag{2}$$

where a is the apparent area of the observed region and k the constant of Stefan's law (in suitable units).

Let us now assume that the reflected solar energy is unaltered in quality, i.e., that the planet's albedo for all wave-lengths is equal to the visual albedo A. We may set $A=pq$ where q depends only on the law of phase variation and p is the ratio of the radiation reflected at full phase to that emitted by a self-luminous body of the

[1] This factor probably varies somewhat with the air-mass traversed by the sun's rays. Such variations can be taken into account by small changes in t'. Observations bearing on this are especially to be desired.

same size and position, which radiates as much energy from each unit of its surface as the planet receives from the sun under normal incidence.[1]

Taking the solar constant as 1.932 calories per square centimeter per minute,[2] and the constant of Stefan's law as 5.72×10^{-5} ergs[3] or 8.21×10^{-11} calories per square centimeter per minute, it is readily found that the energy reflected from a surface for which $p=1$, at a distance of R astronomical units from the sun, is equal in amount (though not in quality) to that emitted from a black surface at the temperature T_0 where

$$T_0=392^\circ R^{-\frac{1}{2}}. \tag{3}$$

If b is the whole area of the planet's disk (including the unilluminated portion), the energy reflected at full phase will be $kbpT_0^4$ and at any phase ϕ times that at the full or $kbA\phi T_0^4/q$, where ϕ is less than 1 and depends on the phase-angle α. Finally, if x is the ratio of the mean apparent surface brightness of the observed area to that of the whole disk, S will be xa/b times the quantity just found, or

$$S=kax\frac{A\phi}{q}T_0^4. \tag{4}$$

From equations (1) to (4) we have

$$\frac{tq\epsilon}{t'\phi Ax}\frac{T^4}{T_0^4}=\frac{0.695}{W}-1. \tag{5}$$

The factors q, A, and ϕ are known;[4] x can be determined by observation for any given area (though at present very little is known on this matter). It may be much greater than unity for a bright spot or for the illuminated limb of a crescent, or much less for a dark spot or the terminator. For the planet as a whole $x=1$, by definition. The emissivity ϵ can only be conjectured. For a planet with a deep atmosphere it is likely to be near unity, and T will in this case be the temperature of the "effective radiating layer." Any uncertainty regarding values of A for visual light and total radiation can be taken up by changing ϵ. It is therefore permissible, formally, to consider values of ϵ somewhat exceeding unity.

[1] Russell, *Astrophysical Journal*, **43**, 178, 1916.
[2] Abbot.
[3] Coblentz.
[4] Russell, *op. cit.*, pp. 179, 190.

The theoretical temperature of the surface, due to solar radiation alone, under idealized conditions, may be computed by the equation

$$T'^4=\frac{y(1-A)}{\epsilon}T_0^4, \tag{6}$$

Where ϵ is the emissivity of the surface, A its albedo, and y is the ratio of the mean rate of radiation from the area considered to the rate in equilibrium under a vertical sun. For the center of the full moon y is probably near unity. For a perfectly conducting sphere $y=\frac{1}{4}$. For a point at latitude L on a planet rotating so rapidly that there is no sensible diurnal inequality of temperature, $y=\frac{\cos L}{\pi}$ (provided that there is no convection of heat from one part of the planet's surface to another). On an actual planet y will be greater than this by day and less by night, by an amount increasing with the rotation period, and may be greater in the afternoon than in the morning.

2. *Transmissions.*—The transmission coefficients t and t' now remain to be considered. The depletion in the earth's atmosphere is different for long and short wave-length radiation and must be allowed for in detail.

From the work of Abbot and Fowle[1] there is a wealth of material available for determining the value of t' for the solar radiation. The mean transmission of solar energy for Washington (altitude 0) is 0.701 for unit air-mass, with a factor of 0.797 for each additional air-mass; and the corresponding values for Mount Wilson (altitude 1800 meters) are 0.819 and 0.895. It may therefore be estimated that at Flagstaff (altitude 2200) and at Mount Hamilton (altitude 1280), the respective transmissions are 0.83 and 0.80, and the air-mass factors 0.91 and 0.88. These coefficients vary slightly with the amount of water-vapor present, but, under the actual conditions of observation, this variation is unimportant.

The amount of precipitable water, Q, expressed in centimeters, in the path of the radiation, is derived from an empirical formula,[2]

$$Q=mhe, \tag{7}$$

[1] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, Vols. II and III.

[2] Fowle, *Astrophysical Journal*, **37**, 359, 1913.

where m is the air-mass determined from the secant law, e the pressure of aqueous vapor in centimeters at the surface, and h a factor depending on the altitude which, according to Fowle's formula, is 1.6 for Flagstaff and 1.8 for Mount Hamilton. Fowle regards these formulae, which represent mean conditions fairly well, as of little value for individual days. From Mount Wilson experience the actual values are as likely as not to lie between half and twice those given by the formula, the reason being that the available humidity observations apply only to the air near the earth's surface. In the future a spectroscopic determination of the total water-vapor should be an important part of radiometric measurements.

To obtain t, black-body energy curves were drawn for different temperatures, integrated by mechanical quadratures, and Fowle's[1] values of the transmissions applied to the successive wave-lengths, and also modified for selective absorption and reflection by the fluorite window of the thermopile (which values were kindly furnished by Dr. Coblentz). The results were finally multiplied by 1.1 to allow for the more complete reflection of these long waves than the solar energy by the silvered mirrors of the telescope. Such corrections, strictly speaking, should be applied to both t and t'; but as only their ratio appears in equation (4), it suffices to apply a differential correction to t alone. This factor is rather uncertain, depending on the condition of the mirrors, and, if necessary, could be determined experimentally. The final values for the planetary transmissions for unit air-mass are given in Table I.

They were originally worked out for different amounts of precipitable water and for each air-mass separately. It was then found that the resulting values could be represented, with an average error of one part in one hundred and fifty, by taking the transmission for air-mass 1 and the amount of precipitable water *actually traversed* by the radiation and multiplying this by the factors 0.96, 0.93, or 0.89 for air-masses 2, 3, and 4, respectively. This convenient approximation cannot be applied if the precipitable water along the path is less than about 0.5 cm. For larger amounts, the absorption, even for rays from the zenith, is already nearly complete wherever

[1] Fowle, *Water Vapor Transparency to Low Temperature Radiation*, "Smithsonian Miscellaneous Collections," Vol. **68**, No. 8; *Smithsonian Physical Tables* (7th ed.), p. 308.

it takes place at all. A fairly dry atmosphere (0.5 cm to 4 cm precipitable water) acts as a very efficient "color screen" for long waves, transmitting nearly all the radiation between 9μ and 12μ, and hardly anything else. It is fortunate that this is the range of humidity likely to be met with at the best observing stations. Under these conditions the transmission falls off rapidly with the temperature of the planet, so that the energy received at the thermopile varies as the sixth or seventh power of the latter. These circumstances are really very favorable for the determination of planetary temperatures.

TABLE I

TRANSMISSION OF BLACK-BODY RADIATION

Total Precipitable Water in Path	Temperature						
	100°	150°	200°	250°	300°	350°	400°
0.003 cm.....	0.002	0.025	0.084	0.170	0.268	0.355	0.440
0.03 cm.....	0.001	0.020	0.073	0.150	0.222	0.288	0.342
0.5 cm.....	0.000	0.014	0.050	0.097	0.136	0.168	0.192
1.0 cm.....	0.000	0.013	0.048	0.091	0.126	0.152	0.171
2.0 cm.....	0.000	0.013	0.047	0.089	0.122	0.147	0.163
3.0 cm.....	0.000	0.013	0.045	0.088	0.120	0.142	0.158

There is, however, great need for an extension of the existing data on the transparency of water-vapor to long-wave radiation. Most of the coefficients used for computing Table I were obtained by more or less free extrapolation and should be checked by experiment.

3. *The observations.*—The first measures of water-cell transmissions were made at the Lick Observatory[1] in 1914 by Dr. Coblentz, and the more recent ones at Flagstaff by Drs. Coblentz and Lampland[2]—to whom the writer is much indebted for communication of results in advance of detailed publication.

Table II contains the observational data and the computed temperatures. W is the water-cell transmission, m the air-mass, Q the precipitable water in centimeters along the path, a the phase-angle, and A the albedo. q/ϕ is the phase factor for reflected radia-

[1] *Lick Observatory Bulletin*, No. 266.

[2] Coblentz, *Scientific Papers of the Bureau of Standards*, No. 438; Coblentz and Lampland, *Popular Astronomy*, **30**, 551, 1922.

tion and x is the brightness factor defined in §1. T_0 is the black-disk equilibrium temperature from equation (3), while T'_1 and T'_2 are theoretical temperatures computed according to equation (6), the former for a rapidly rotating planet, in which case $y=\frac{\cos L}{\pi}$, and the latter for the sun permanently on the meridian, $y=\cos L$. If the planet's radiating layer is black for long waves ($\epsilon=1$), its tem-

TABLE II

PLANETARY TEMPERATURES

Planet	Date	W	m	Q	a	$\frac{q}{\phi}$	A	x	T_0	T'_1	T'_2	T
Venus	1914, Aug. 19	0.59	5.5	4.7	67°	3.03	0.59	2	460°	276°	369°	330°
	1922, June 15	0.663	3.6	2.0	45	2.16	.....	3	.....	.....	.....	310
Mars	1921, Oct. 6	0.67	2.6	2.0*	15	1.30	0.154	1	304	220	293	.....
Equator	1922, June 15	0.470	2.1	1.2	5	1.17	.....	.....	323	233	310	268
South	1922, June 15	0.489	.....	.....	.....	.....	.....	.....	.....	227	303	262
North	1922, June 15	0.511	.....	.....	.....	.....	.....	.....	.....	223	297	255
Equator	1922, June 18	0.476	2.1	1.8	8	1.20	.....	.....	.....	233	310	264
South	1922, June 18	0.523	.....	.....	.....	.....	.....	.....	.....	227	303	252
North	1922, June 18	0.551	.....	.....	.....	.....	.....	.....	.....	223	297	242
Jupiter	1914, Aug. 17	0.657	1.7	0.7	2	1.5	0.56	1	174	107	142	157
	1922, June 14	0.682	2.2	1.1	10	.....	.....	.....	168	103	137	168
Saturn	1914, Aug. 26	0.55	1.6	0.7	0	1.5	0.63	1	130	76	101	172
	1922, June 14	0.60	1.5	0.7	.....	.....	.....	.....	127	75	100	161
Moon	1914, Aug. 27	0.147	3.6	2.9	80	1†	0.073	1†	392	290	386	400
Earth	.....	.....	.....	.....	.....	.....	0.45	.....	392	254	338	.....

* No humidity data; a guess.

† $\frac{q}{\phi \cdot x}$ assumed equal to 1.

perature must lie between these limits. With lower emissivity these computed limits will be multiplied by the factor $\epsilon^{-\frac{1}{4}}$. The observed temperature, T, will be multiplied by a factor approximating $\epsilon^{-\frac{1}{4}}$ (see §2). T is the temperature computed from the water-cell transmissions by equation (5), assuming that the emissivity $\epsilon=1$. The computed values for the earth are inserted for the sake of comparison. It should be remembered that for planets with an atmosphere, T represents the temperature of the "effective radiating layer." The surface temperature may be considerably higher.[1]

DISCUSSION

Venus.—The quantity x was estimated on the basis of a sketch sent by Dr. Coblentz, showing the region observed. Were it not for the uncertainty in this, it might be possible to judge from the obser-

[1] Compare the discussion by Milne, *Philosophical Magazine*, **44**, 872, 1922.

vations whether the period of rotation is long or short. The table shows that the observed temperatures agree fairly well with those computed for a non-rotating or very slowly rotating planet, and as x in the second case may be greater than 3, the agreement may be even better. More data are needed before a decision can be made. Observations of the radiation from small areas of the bright crescent and the dark side of the planet would be of special value, and might show a difference between the sunrise and the sunset terminators.

Mars.—The first isolated observation is very discordant.[1] In the absence of data regarding humidity this has been guessed at, but no reasonable change in the estimate will remove the discordance. The more recent observations give a mean temperature of 257° A. or −16 °C., which is surprisingly high. It may perhaps be explained by assuming that the surface is very little shielded by the atmosphere, and has a low-heat capacity, so that it reaches nearly the equilibrium temperature T'_2 in the early afternoon, cooling greatly at night. A nocturnal fall of 60° C. would be required to maintain the proper average diurnal radiation. The differences between the equator and the higher latitudes are very closely what might be expected, as the sun was 3° south of the Martian equator. The effective latitudes of the observed northern and southern regions have therefore been taken as 33° and 27° in computing T'.

Jupiter and Saturn appear to possess nearly the same temperatures which, although very low, −110° C., are much higher than can be explained by the solar heat alone. They are far from red hot on the surface (as has been often supposed), but radiate about as much heat as if the uppermost layers of the clouds upon their surfaces were composed of carbon dioxide snow, or some similar substance.

The moon.—The observed portion was close to the limb and in a region of very unequal brightness. The area covered by the thermopile is so small that x can hardly be guessed at. It was assumed that $q/\phi x = 1$ (the value for a part of the full moon of albedo equal to the average). The resulting temperature, 127° C., agrees very well with the calculated value.

[1] In a letter Dr. Coblentz states that the seeing conditions were very poor, and that a transmission of 60 per cent would be just as probable as the published 67 per cent. The observed temperature is therefore purposely omitted from the table.

It would be well worth while to observe the moon with a small reflector, using the entire disk on the receiver of the thermopile, and also to determine, with a larger telescope, the distribution of the heat over the surface. It should then be possible to obtain very accurate results. Professor Very[1] in 1889 made a number of observations of the heat distribution on the moon, using glass instead of a water cell to isolate the long-wave energy. An examination of these results shows them to be in agreement with those of this paper (viz., that the moon is very hot), within the accuracy possible at that time. He finds that the brighter parts of the moon's disk show a greater water-cell transmission than the darker areas (as they should do), and that the isothermal curves have nearly the theoretical distribution, approximating circles perpendicular to the line joining the center of the disk with the sun.

4. *Further suggested observations.*—Of the planets which have not yet been observed, Mercury should give a very low water-cell transmission on account of its low albedo and high temperature. Observations would be interesting but difficult.

Uranus should be observable. If it is as hot as Jupiter and Saturn, it should be possible to determine its temperatures with some accuracy. Neptune may be accessible with great telescopes.

SUMMARY

Theoretical conclusions.—The results obtained so far are encouraging and indicate that the method of water-cell transmissions affords a very valuable means of determining planetary temperatures.

The absorption of radiation in the earth's atmosphere does not apparently offer any serious obstacle although a more accurate determination of the absorption of water-vapor for long waves is desirable.

Extending the observations on the planets over widely varying zenith distances will largely eliminate the uncertainty of the water-vapor correction.

To obtain a reliable value of the mean temperature, it is necessary to use a receiver large enough to cover the entire disk.

[1] Very, *The Distribution of the Moon's Heat and Its Variation with Phase*, published by Utrecht Society of Arts and Sciences, 1891; Very, *Astrophysical Journal*, **8**, 199 and 265, 1898.

Additional smaller ones are also desirable for the study of the heat distribution over the planet's surface. Either fluorite or rock-salt should be used for the window of the thermocouple.

Numerical conclusions.—Coblentz' observations have been reduced, and indicate that the surface temperature of Venus is 50° C., Mars −16° C., Jupiter and Saturn −110° C., while the moon is very hot, possibly 120° C. These values are subject to some uncertainty, especially the first and last, owing to the difficulty of estimating the correction which has to be applied because only a part of the image fell upon the thermopile.

The computed temperatures of Venus and Mars can be explained if they receive all their heat from the sun and lose it almost as fast as they receive it, cooling greatly at night.

Jupiter and Saturn have higher temperatures than could be maintained by solar radiation alone, confirming the belief that the planets are hot internally.

Further observation on all accessible members of the solar system is highly desirable.

Acknowledgments.—In conclusion, I wish to express my appreciation to Professor Russell, to whom I am indebted, not only for the first section of this paper, but for many conferences concerning the entire problem. I wish also to express my gratitude to Drs. Coblentz and Lampland, who kindly furnished me with all the necessary data in advance of publication; to Dr. Fowle for advice concerning the air transmissions, and to Professor Very for the communication of his early measures of planetary radiation.

PRINCETON UNIVERSITY
February 1923

LIGHT-CURVE AND ORBIT OF CG CYGNI

$\alpha = 20^h 54^m.2$; $\delta = +34^\circ\ 47'.5 (1900.0)$

By CH'ING-SUNG YÜ

ABSTRACT

Photometric study of the eclipsing variable star CG Cygni.—The *light-curve* of this variable star is determined from a series of 272 extra-focal observations made with the Thaw Photographic Refractor of the Allegheny Observatory during 1922 (Fig. 2). It is found to be an eclipsing system with two minima, the eclipses being partial. *Ellipticity* of the component stars is indicated by the continuous variation between eclipses; there is also definite indication that the system is slightly brighter when the brighter component is approaching us than when it is receding from us. The *range of variation* (photographic) is 1.12 mag.; the *period* is 0.631,139 day. The *orbital elements* and probable dimensions of the system were determined on the assumption of uniform illumination and also on the hypothesis that darkening at the limb exists (Table V and Figs. 4 and 5). The latter solution gives the smaller residuals, the probable error of a normal of weight unity being 0.019 mag. for the "uniform" and 0.014 mag. for the "darkened" solution. The larger and fainter component of the system is of later spectral type than the smaller and brighter star.

Among the variable stars included in the extensive photometric program now being carried on at the Allegheny Observatory was the recently discovered variable star B.D. $+34^\circ$ 4217, which has now received the name "CG Cygni." The variability of this star was discovered by A. Stanley Williams in the year 1905, when comparing two photographs. Observations to determine its elements of variation were not commenced however until October, 1921, when the discoverer made a series of 219 visual observations.[1] While the shape and time of the principal minimum were well determined, observations were too few to represent the true form of other parts of the light-curve, especially the secondary minimum. Williams' results are given in Table II.

Four stars in the neighborhood of the variable were selected as comparison stars. They are designated *a*, *b*, *c*, and *d* in the order of decreasing brightness, and their positions relative to the variable can be seen in Figure 1. Their positions for 1900 with their magnitude differences and the adopted magnitudes are given in Table I. The positions are not exact, but will serve for identification.

For the observations eleven sets of plates were taken on eleven nights with the 30-inch Thaw Photographic Refractor set about

[1] *Monthly Notices*, **82**, 300, 1922.

10 mm out of focus. Each set consists of four "Seed 30" plates. Following the usual procedure of the photometric work at the

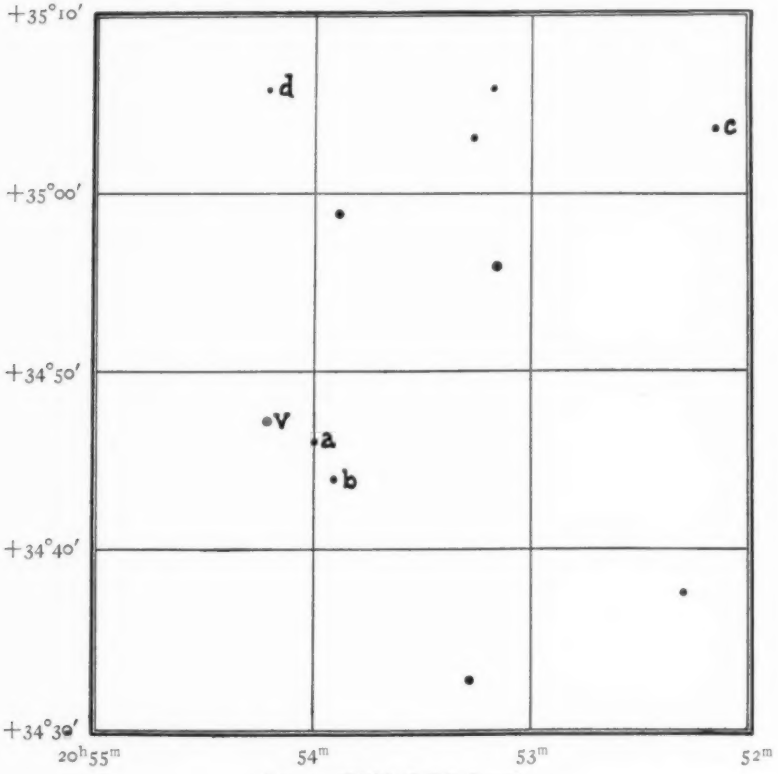


FIG. 1.—Field of CG Cygni

TABLE I

POSITIONS AND MAGNITUDES OF COMPARISON STARS

Star	α	δ	Mag.-Diff.	Adopted Mag.
a	20h54m0	+34° 46′	0.000	10.100
b	20 53.9	+34 44	0.313	10.413
c	20 52.2	+35 04	0.327	10.740
d	20 54.2	+35 06	0.381	11.121

Allegheny Observatory, the four plates were exposed in succession to one setting of the star; then the position of the star was changed about 2 mm and the process repeated. The average length of a

series of observations was about five hours, which covered a considerable portion of the period of the variable. The time of each exposure ranged from five to eight minutes, depending upon the clearness of the sky. With the exception of passing clouds on a few occasions all plates were taken under favorable conditions. However, it was deemed advisable later to reject one set of plates which were taken on a partially cloudy night; there were only one or two exposures on each plate of this set. The other ten sets of plates were used, making a total of 272 observations, which cover the period at least twice and the minima three or four times.

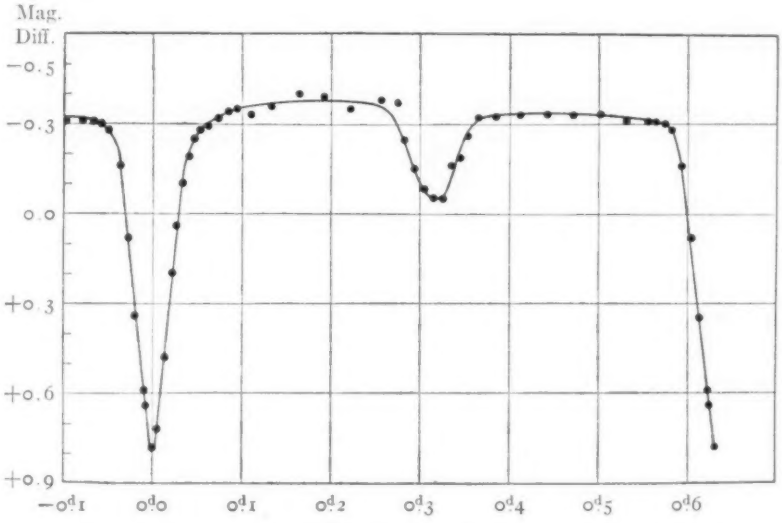


FIG. 2.—Mean light-curve of CG Cygni

The intensities of the star images were measured with a Hartmann microphotometer. Each reading on the variable was compared with the mean s of the comparison stars, and the magnitude-differences $(v\text{-}s)$ obtained. These observations were grouped into forty-four normals by taking the means of every 0.005 day during principal minimum, every 0.010 day during secondary minimum, and every 0.030 day between eclipses. The phases have been reduced to the sun. The normals are entered in the second and third columns of Table IV, from which the mean light-curve (Fig. 2) is derived.

By making a preliminary plot of the individual observations, three well-defined minima were found. These indicated that the period is a little shorter than that given by Williams. To fit these, the period as determined by Williams was corrected and slightly shortened, using the original value of the epoch. The new period adopted, and other elements of variation derived from the mean light-curve, are given in Table II, together with the corresponding values obtained visually by Williams.

TABLE II

ELEMENTS OF VARIATION OF CG CYGNI

	Williams (vis.)	Yü (phot.)
Period	$0^d.63118$	$0^d.631139$
Principal minimum	$10^h.42$	$11^h.374$
First maximum	9.93	10.219
Secondary minimum	10.15	10.544
Second maximum	9.94	10.264
Range	0.49	1.115

The adopted formula for phase determination is:

Helio. G.M.T. of Principal Min. = J.D. 2,422,967.4268 + $0^d.631139$ E.

It will be seen by comparing the two sets of results in Table II that a visual range of but 0.49 mag. was found, whereas the corresponding range as found from the photographic data is 1.115 mag. While such differences might be taken as indicating a color-index of roughly +0.3 for CG Cygni, such a deduction is highly uncertain, and the determination of the color-index of this variable must await future color-screen comparisons. The color-index seems definitely greater at primary minimum.

The mean light-curve of CG Cygni (Fig. 2) shows that the system is an eclipsing binary of the "β Lyrae" type, the eclipses being partial. There is a deep primary minimum and a shallower secondary minimum. The former indicates a loss of about 0.6 and the latter about 0.2 of the light of the system. The duration of each partial eclipse is about 0.1 day. The secondary eclipse occurs exactly midway between the primary eclipses, this, together with the equality in the duration of the eclipses, indicates that the orbit

is sensibly circular. Its eccentricity has therefore been assumed as zero ($e=0$). The slight but constant variation of light between eclipses shows that the component stars are elliptical to a small extent. Mutual heating and reflection effects are also indicated to an appreciable amount, as the system is brighter near the secondary eclipse than near the primary. One interesting fact definitely shown by the mean light-curve is that the system is decidedly brighter following the primary minimum than after the secondary minimum, although the positions of the two stars are symmetrically identical in both cases. Since the orbit is sensibly circular, this phenomenon cannot be accounted for by a periastron effect. It will be noticed that the system appears brighter when the bright star is approaching us, and is slightly fainter when the same star is receding from us. Hence it has, in this respect, the characteristics of a Cepheid variable and the "resisting medium" would perhaps explain this discrepancy. There is the difficulty on this theory, however, due to the fact that no effect is observed, when the larger and fainter star is approaching at the same velocity. It will be noticed further that the observations along the first maximum do not fit the smooth curve so well as those along the second maximum do, although more observations were made during the former. The curve at the latter phase is remarkably smooth in spite of the fact that it was covered only twice by observations. The apparently unsteady character during the first maximum is perhaps real, as a result of increased activity of some sort, although errors in observation may account for most of it.

The mean light-curve is "rectified" for the effects discussed above, namely, ellipticity, reflection or radiation effect, and the "resisting medium" effect. The first was corrected by Russell's graphical method, and the corrections (m_e) entered in Table III. The other two effects were small enough to warrant summary treatment and are simply removed by $(-.02 \cos\theta + .022 \sin\theta)$. The rectified magnitudes,

$$M = m - m_e - .02 \cos\theta + .022 \sin\theta$$

are given in the fourth column of Table IV, and from these values the rectified curve (Fig. 3) is derived.

The rectified light-curve shows the normal characteristics of the "Algol" type. The drop of the principal minimum is 1.061 mag., and that of the secondary minimum 0.271 mag. The loss of light for the two minima is as follows: $1-\lambda_1=0.624$ and $1-\lambda_2=0.221$.

TABLE III

CORRECTIONS FOR ELLIPTICITY

θ	t	$0.100\cos^2\theta$	m_e
0°	.000	0.100	0.057
15	.026	.093	.053
30	.053	.075	.042
45	.078	.050	.028
60	.105	.025	.014
75	.132	.007	.004
90	.158	0.000	0.000

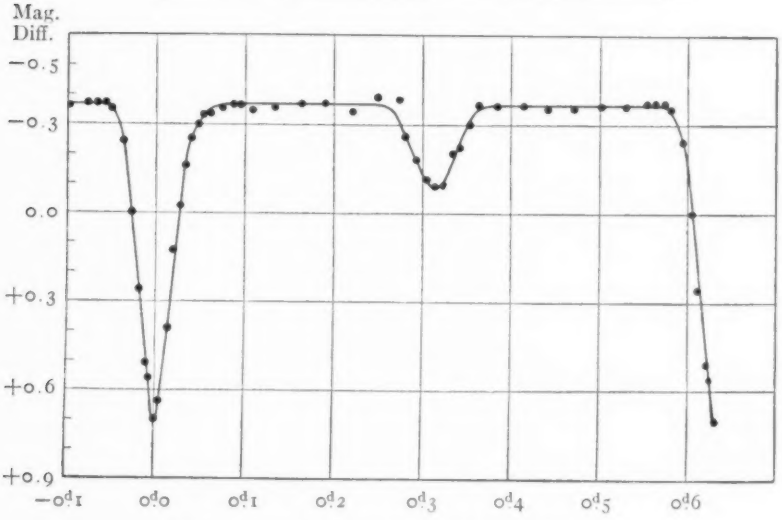


FIG. 3.—Rectified light-curve of CG Cygni

In deriving the orbital elements of CG Cygni, the writer applied the principles of the eclipse theory, as developed by Professor H. N. Russell.[1] Both "uniform" and "darkened" solutions were made. The results are given in Table V, and the system represented in Figures 4 and 5.

[1] *Astrophysical Journal*, **35**, 315; **36**, 239, 1912.

TABLE IV

NORMAL POINTS, RECTIFIED MAGNITUDES, AND RESIDUALS

No. of Normal Points	Phase t	Mag. $(v-s)$	Corr.	Rectified Mag.	Wts.	$O-C_u$	$O-C_d$
1.....	$0^d.000$	+0.78	−0.08	+0.70	3	.00	.00
2.....	.005	+0.72	−0.08	+0.64	3	+.01	.00
3.....	.014	+0.48	−0.07	+0.41	6	+.03	+.02
4.....	.021	+0.20	−0.07	+0.13	3	−.05	−.06
5.....	.028	+0.04	−0.06	−0.02	5	−.05	−.05
6.....	.034	−0.10	−0.06	−0.16	4	−.06	−.06
7.....	.041	−0.19	−0.05	−0.24	7	−.06	−.04
8.....	.048	−0.25	−0.05	−0.30	3	−.05	−.02
9.....	.053	−0.28	−0.05	−0.33	2	−.05	−.02
10.....	.062	−0.29	−0.04	−0.33	8	−.01	.00
11.....	.075	−0.32	−0.03	−0.35	6	+.01	+.01
12.....	.085	−0.34	−0.02	−0.36	4	.00	.00
13.....	.095	−0.35	−0.01	−0.36	8	.00	.00
14.....	.110	−0.33	0.00	−0.33	12	+.03	+.03
15.....	.134	−0.36	+0.01	−0.35	20	+.01	+.01
16.....	.104	−0.40	+0.02	−0.38	12	−.02	−.02
17.....	.192	−0.39	+0.02	−0.37	10	−.01	−.01
18.....	.223	−0.35	+0.01	−0.34	9	+.02	+.02
19.....	.256	−0.38	−0.01	−0.39	10	−.04	−.03
20.....	.274	−0.36	−0.02	−0.38	2	−.08	−.07
21.....	.282	−0.24	−0.02	−0.26	3	+.01	.00
22.....	.295	−0.15	−0.03	−0.18	4	+.01	.00
23.....	.304	−0.08	−0.03	−0.11	4	+.02	+.01
24.....	.314	−0.05	−0.04	−0.09	5	.00	.00
25.....	.324	−0.05	−0.04	−0.09	3	+.01	.00
26.....	.334	−0.16	−0.04	−0.20	4	−.02	−.03
27.....	.344	−0.18	−0.04	−0.22	3	+.01	+.01
28.....	.354	−0.26	−0.04	−0.30	4	−.01	.00
29.....	.364	−0.32	−0.04	−0.36	3	−.04	−.02
30.....	.385	−0.32	−0.03	−0.35	10	−.01	−.01
31.....	.413	−0.33	−0.03	−0.36	14	.00	.00
32.....	.443	−0.33	−0.02	−0.35	10	+.01	+.01
33.....	.473	−0.33	−0.02	−0.35	16	+.01	+.01
34.....	.503	−0.33	−0.03	−0.36	13	.00	.00
35.....	.532	−0.31	−0.05	−0.36	13	.00	.00
36.....	.555	−0.31	−0.06	−0.37	4	−.01	−.01
37.....	.564	−0.31	−0.06	−0.37	3	−.02	−.01
38.....	.574	−0.30	−0.07	−0.37	2	−.06	−.05
39.....	.582	−0.28	−0.07	−0.35	2	−.08	−.06
40.....	.595	−0.16	−0.08	−0.24	3	−.09	−.07
41.....	.605	+0.08	−0.08	0.00	3	−.05	−.06
42.....	.613	+0.34	−0.08	+0.26	3	+.03	+.03
43.....	.622	+0.59	−0.08	+0.51	3	−.02	−.03
44.....	.626	+0.64	−0.08	+0.56	2	−.06	−.06
1.....	0.631	+0.78	−0.08	+0.70	3	.00	.00

While it is not the primary purpose of this paper to present the relative merits of the "uniform" and the "darkened" hypotheses of the eclipsing theory of variable stars, it would seem from the results obtained that darkening at the limb is strongly indicated for CG Cygni. The average residual, regardless of sign, is 0.016 mag. for the darkened hypothesis as against 0.019 mag. for the uniform one. Securing the "probable error" of a single normal of weight unity from the final residuals by the ordinary formula, this value for the uniform solution is ±0.019 mag., and for the darkened

TABLE V

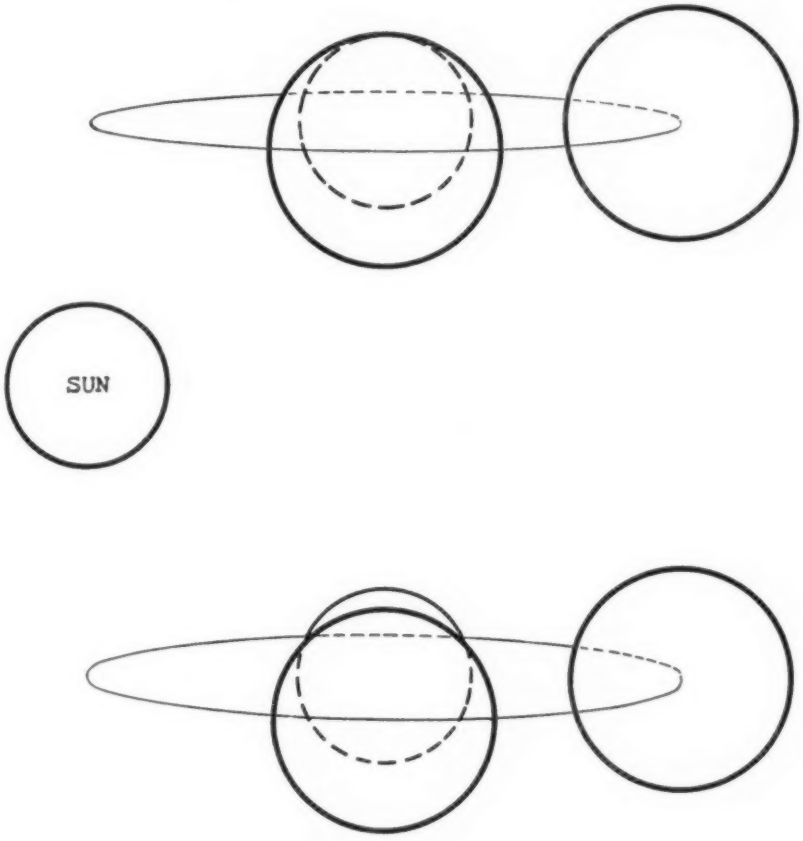
ORBITAL ELEMENTS OF CG CYGNI

		Uniform	Darkened
Ratio of radii of stars	k	.767	.784
Maximum percentage of eclipse	a_0	1.000	.950
Major semi-axis of larger star	a_1	.381	.370
Minor semi-axis of larger star	b_1	.361	.357
Major semi-axis of smaller star	a_2	.292	.290
Minor semi-axis of smaller star	b_2	.277	.280
Eccentricity of meridian section	ϵ	.317	.253
Eccentricity of orbit	e	.000	.000
Inclination of orbit plane	i	85°10′	82°17′
Least apparent distance of centers	$\cos i$	.084	.143
Light of larger star	L_1	.376	.343
Light of smaller star	L_2	.624	.657
Ratio of mean surface intensities	J_1/J_2	1/2.8	1/3.1
Density of larger star (sun = 1)	ρ_1	.340	.358
Density of smaller star	ρ_2	.753	.742
Mean radius of larger star (sun's $r = 1$)	$\bar{r}_1$	1.433	1.408
Mean radius of smaller star	$\bar{r}_2$	1.099	1.015
Hypothetical parallax of system	π	0".0044	0".0044

one, ±0.014 mag. The observed secondary minimum is also much better represented by the "darkened" solution.

It was found, by assuming equal masses for the component stars, that the mean radius of the larger star is about one and four-tenths that of the sun. Assuming that its surface brightness is the same as the sun's, the light-emission of the larger star will be about twice as great, and that of the whole system about 5.7 times as great as that of the sun. At maximum the photographic magnitude of the system is 10.22, and the visual magnitude close to 9.9. Taking the absolute magnitude of the sun as +5, and assuming that the system is 5.7 times as bright as the sun, its absolute magnitude will be about

+3. Then from the equation $M = m + 5 + 5 \log \pi$, we derive a hypothetical parallax of 0".0044, corresponding to a distance of 740 light-years. The distance thus derived is probably trustworthy, though it must be remembered that it depends on the assumptions made above as to mass and intrinsic surface luminosity.



FIGS. 4 and 5.—System of CG Cygni. The upper cut is for the uniform solution; the lower for the darkened solution.

CG Cygni is too faint for spectroscopic study with existing instruments. The larger color-index at primary minimum would indicate that the spectrum of the fainter star is of a more advanced type than that of the brighter star.

SUMMARY

1. Visual observations of the variable star CG Cygni, amounting to 219, were made by Williams, and its epoch and period determined.

2. The writer has taken 272 photographic observations, the reduction of which gives the following new period, the epoch being that given by Williams:

Principal Min. = J.D. 2,422,967.4268 + $0^d.6311139$ E G.H.M.T.

3. The mean light-curve shows that CG Cygni is an eclipsing system of the "β Lyrae" type, the eclipses being partial.

4. At primary minimum when over nine-tenths of the smaller and brighter component is covered by the larger and fainter one, the star is 1.115 mag. fainter than at maximum. The magnitude at maximum is 10.219.

5. The spectrum of the system is unknown, but there is definite indication that the fainter star is later in type than the primary.

6. The component stars are elliptical, but the ellipticity is small. The orbit is sensibly circular.

7. Mutual heating and reflection effects are noticeable. The system is 0.045 mag. brighter at first maximum when the bright star is approaching us than at second maximum when the same star is receding.

8. Two methods of solution were employed to determine the orbital elements of the system. The agreement of the computed curves in each case with the observed one proves that the light-variation of CG Cygni is satisfied by the eclipse theory, and the slightly better representation of the "darkened" solution indicates that darkening at the limb exists.

9. The smaller star emits twice as much light as the larger one, while its surface brightness is 3.1 times as great. The fainter star exceeds the brighter one in linear dimensions by 27 per cent. The distance between centers is 2.7 times the radius of the fainter star. The larger and the smaller components are, respectively, .358 and .742 as dense as the sun on the equal-mass assumption.

10. CG Cygni is probably 740 light-years away, having a parallax of $0''.0044$.

ACKNOWLEDGMENTS

I wish to express my indebtedness to Dr. H. D. Curtis, director of the Allegheny Observatory, who suggested this investigation as a thesis in partial fulfilment of the requirements for the degree of Master of Science at the University of Pittsburgh, placed at my disposal the 30-inch Thaw Refractor and all other instruments and supplies that made the undertaking of this investigation possible, and looked over part of my computed materials. My thanks are also due to Dr. F. C. Jordan, who supervised and directed the work, set aside for me about a dozen nights from his busy photometric schedule, and also checked part of my computations. I am indebted also to Mr. Z. Daniel for photographing a set of plates for me.

ALLEGHENY OBSERVATORY
PITTSBURGH, PA.
February 1923

EXCITATION STAGES IN THE OPEN ARC-LIGHT SPECTRA

PART III. LEAD, MERCURY, THALLIUM, MAGNESIUM

By B. E. MOORE

ABSTRACT

Excitation stages in low-current arc spectra of Pb, Hg, Tl, and Mg.—In the present study *photographic methods* were used in contrast to visual methods of the previous studies. It is found that the *true arc* begins to discharge from a minute point or negative polar tip. This tip discharge never forms without ionization; and when the pole is a bivalent element there are always lines belonging to the Rydberg 4N limit, lines which are theoretically associated with a return of the second valence electron to the nucleus, i.e., at the pole tip, the temperature and velocities of the electrons are always high enough to produce appreciable *dissociation* of the atom. For the element *lead* all of the observed lines are easily produced and show an exceptionally low ratio of pole to middle intensity. This indicates that the lines belong to Stage I and that there is an appreciable amount of lead vapor in the arc at low currents. Two stages are found in the development of the lines of *mercury* and a possible third stage is suggested; furthermore, the lines develop more slowly than in the other metals. Two stages are found in the development of the lines of *thallium*. There are four stages in the development of the lines of *magnesium*, which correspond closely to what is known from other considerations as a resonance, first ionization, a subsequent resonance, and a second ionization. The changes are subtle, and the stages overlap. The different actions of nitrogen and hydrogen atmospheres are used in these experiments. *Enhancements* are carefully considered, and evidence is given which confirms the opinion, which has been frequently contested, that the enhanced lines correspond to hotter stages.

INTRODUCTION

In Parts I and II[1] the intensity of various spectral lines was generally computed from the formula

$$\log_{10} I = d, \qquad (1)$$

where I is the intensity of illumination and d the density of the filter which reduces the transmitted light to the limit of visibility. This method is employed herein for the visible spectrum of mercury (see graphs, Fig. 2). In the present experiments the photographic method has been chiefly employed. If we attempt to use the density of a (negative) line as an index of intensity according to equation (1), we are subject to some restrictions. I am indebted to Mr. Lloyd Jones, of the Eastman Kodak Company, for an extensive discussion of this point. For the present purpose Figure 1

[1] *Astrophysical Journal*, **54**, 191, 246, 1921.

will give us sufficient information. Here logarithms of intensity are represented as abscissae and the corresponding densities of the photographic image as ordinates. The two curves correspond to different wave-lengths. Uniform results are obtained only when lines are studied upon one branch (e.g., branch *b*). A method was available which permitted the study of spectral lines by an absorption method, and from the absorption a computation of the density of the lines. A less accurate method was judged sufficiently satisfactory for the purpose. Let us take, for example, three lines, *l*, *m*, *n*, whose intensities are 100, 10, and 1, respectively. Let us take three exposures of these lines, and let the time ratio of the exposures be 1, 10, and 100. Now we shall make the assumption that an increase in time of exposure is equivalent to an increase in intensity, which certainly is more accurate than the reading which we shall obtain. We find now that *m* in the second has the same density as *l* in the first photograph. Also, we find that *n* in the third photograph has the same intensity as *l* in the first and as *m* in the second. Thus by cross-comparison of differently "timed" exposures, one may make visual comparison of the intensities of lines. This procedure, of course, should be limited to the *b* branch of the curve, Figure 1, and should be subjected to a large correction for large change of wave-lengths. The latter correction has not been introduced, and lines of intensity 1, 2, lie upon the lower branch of the curve, Figure 1. Such lines are essentially brighter, therefore, than they appear.

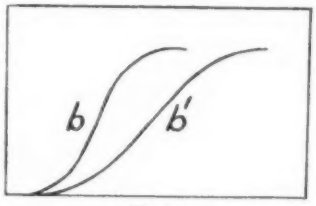


FIG. 1

One other feature should be noticed. If we are dealing with a photograph which has a number of lines whose intensities are distributed across the *b* section of the curve, Figure 1, and compare it with another photograph adequately timed to bring the strong lines down to the lower part of the branch *b*, then the lines which previously occupied the lower position upon the *b* curve will make practically no impression upon the lower branch of the curve. Hence the disappearance of these lines under such circumstances is

not an indication of the absence of the lines in the source of illumination. There are two methods which one may adopt to overcome this difficulty. First, one may establish some relationship between the intensities of certain lines and the current. In Parts I and II we have used

$$I = ki^x, \tag{2}$$

where i is the current, and we found very serviceable values for x to be 2.5, 3 to 3.3. Then with x equal to 3 for two currents of ratio one to two, the times required for exposures which will give equal intensities are in the ratio of eight to one. In this way any inequality in the rate of development will appear. For the very weak lines one may resort to overexposure, the object of which is to push the line from the lower to the middle branch in Figure 1. This is essentially increasing x, and its application is limited by the fogging of the plate from low-intensity continuous radiation.

LEAD

The metal was placed in a small brass cup and was used as a lower negative pole. The upper positive pole was copper. The current was varied from 0.035 ampere to 0.1 ampere. With larger

TABLE I

Group I			Group II*		
λλ	*I*	Negative Pole—Middle	λλ	*I*	Negative Pole—Middle
4058	20	5	2073	5	10
3740	4	10	33	10	10
3680	20	5	23	3	10
3671	1	10	02	5	10
3640	20	5	2663	4	10
3573	3	10	28	8	10
			2577	1	
			11*c*	1	
			2476*a*	1	
			2446*b*	1	

* Groups according to Kayser and Runge, *Wied. Ann.*, **52**, 93, 1894; "*a*, *b*," belong to Group III; "*c*" an unidentified line.

currents the metal volatilized rapidly. The arc was photographed in an atmosphere of nitrogen. In Table I is given a list of lines with an eye estimate of intensities for one current. With increased

exposure the continuous field grows, and, except for a small region about λλ 2400–2800, interferes with observations. In the latter region there was no trace of new lines at 100 times overexposure, although there are several recorded in Kayser's *Handbuch der Spectroscopie*, Volume VI, which might have been anticipated. Lead is characterized by a few very strong lines, a few of moderate intensity, and a greater number of very weak ones. Under the circumstances the very weak ones may have been absent because of inadequate exposure, but this cannot be said concerning the missing lines of moderate intensities. The inference is that the current had not reached sufficient intensity to excite these lines. Comparing the ratio of polar to middle intensities in Table I, it is evident that the slight variation does not give one adequate reason for thinking that the lines belong in different classes. They represent very easy excitations and probably fall in Stage I. They increase more rapidly than the cube of the current.

MERCURY

Table II gives a list of lines at a current of .08 ampere whose terms are listed in order of their relative intensities. A larger current was not used in the invisible spectrum, for boiling and spraying begins. Judgment concerning the middle of the arc could not be carried very far as an exposure forty times as long as at the pole begins to produce a fog which has its origin in continuous spectral radiation.

Table III gives a study of λ 5461 with the ray filters in the visible spectrum. To get the last readings for pole and middle it was necessary to use a large surface of mercury, and to have an assistant stir it and remove the oxidized portion. This line and a few others are shown upon the graph, Figure 2. Line λ 5769, negative pole, practically duplicates λ 5461, middle of the arc. Line λ 5790 is like λ 5769 but a little weaker. Line λ 4916, Type $2P$–$3S$, shows the same ratio to λ 5769 as λ 5769 shows to λ 5461. Lines λ 4339, Type $2P-4d'$ and λ 4347, Type $2P-4D$, are just becoming visible at the limits of the experiment. Line λ 4959 appears at the same time as λ 4916 and shows uniformly about one-third of the intensity of the latter. An unknown type is shown in λ 5365, a very important unknown type is found in Table II, line λ 4078.

TABLE II

MERCURY LINES

λλ	Intensities Negative Pole	Negative Pole—Middle	Type	Stage
2537	100	20	$1S-2p_2$	I
5461	.....	.....	$1.5s_1$	I
4358	100	20	$1.5s_2$	I
4047	60	20	$1.5s_3$	I
3663	10		$3d''_1$	II
55	12		$3d'_1$	II
50	20	30	$3d_1$	II
3131	.....	.....	.....	.....
26	15	30	$3d_2$	II
3028	10	.....	$3d_3$	.....
3028	.....	.....	$4d''_1$	.....
23	.....	.....	$4d_1$	.....
21	15	30	$4d_1$	II
2655, 2653, 2652 *	15	.....	$4d_2$	II
2635†	.....	.....	$4d_3$	II
3341	10	>40	$2.5s_1$	.....
2893	7	.....	$2.5s_2$	.....
2753	4	.....	$2.5s_3$	.....
4078	10	.....	?	.....
2925	6	.....	$3.5s_1$	.....
2574	4	.....	$3.5s_2$	.....
2464	1	.....	$3.5s_3$	.....
2803	7	.....	$5d_1$	.....
2482	2	.....	$5d_2$	.....
2699	4	.....	$6d_1$	.....
2399	1	.....	$6d_2$	.....
5790	g	.....	$2P-3D_1$	.....
5769	g	.....	$2P-3d$	.....
4347	g	.....	$2P-4D$	.....
4339	g	.....	$2P-4d_1$	.....
4916	g	.....	$2P-3.5S$	.....
5335	g	.....	.....	.....
4959	g	.....	.....	.....

* Not separated. Broad. † Overlapped by λ 2537. g See graph.

TABLE III

λ 5461

Intensities	Negative Pole (amperes)	Middle (amperes)
10	.....	.016 −.018
100	.0125	.0295−.0345
1000	.0215−.0245	.056 −.068
10000	.036 −.045	.095 −.125
100000	.11−.16*	.....

* Probably reversal is taking place.

There is a marked degree of parallelism in the lines of the mercury graph, Figure 3. A rate of development proportional to the cube of the current is shown, and it is seen that the lines develop less rapidly than this power of the current. The same conclusion holds for the ultra-violet lines. Line λ 2537, Type $1.5s—2p_2$, and the triplet $2p_1—1.5s$ intercept the ordinate axis at zero current, i.e., they leap to well-defined magnitudes at the very lowest current at

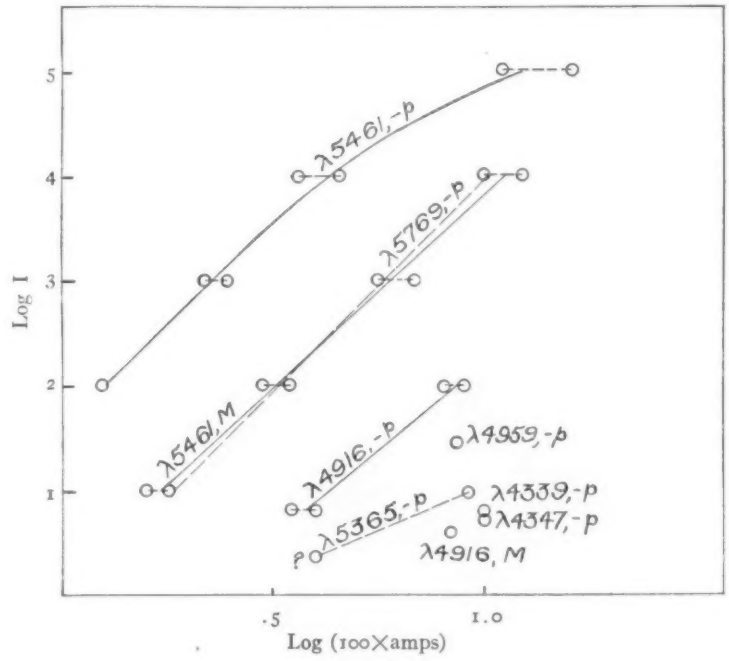


FIG. 2

which the self-sustaining arc will operate. They are therefore placed in Stage I. Then there follow others whose claim for Stage II are very distinct. Lastly there are weaker lines in the ultra-violet and some just becoming distinct in the visible spectrum for the largest current. The evidence that these lines belong in a third stage is not so well defined. But if they belong to an earlier stage, we must assume that the general slope of their development is very different from the other lines in Stage II. The lines are not assigned to any stage in Table II.

THALLIUM

Line λ 5350 is so very strong that it required too much superposition of filters to get a clear view of the field, and readings are too inaccurate to record. However, between 0.08 ampere and 0.32 ampere the rate of development seemed less than the 2.5 power of the current. An alloy of thallium and copper, containing 1.4 per cent of thallium, was tried. This was similar to the recorded line λ 5378 in cadmium but considerable brighter.[1] But at about .1 ampere the samples melted, rapidly oxidized, and the line fell off to about one-tenth its original intensity. With this scarcely complete analysis, it was concluded that the character of λ 5378 had been properly established in the investigation of cadmium, although the identity of the substance was wrong. With the alloy as negative pole, an exposure of the middle of the arc at .08 ampere, 100 times as long as the foregoing negative pole exposure brings out the lines $2s_2$, $3d'_1$, $3d_1$, $3d'_2$, $3d_2$, and the first-stage copper lines λλ 3247, 3274.

From this analysis, these lines are assigned to Stage I. They belong to the earliest ring types immediately following the resonance position and apparently cannot be separated from it in the self-sustaining arc. The remainder of the lines have the Stage II characteristics. They fall off rapidly in intensity as required, and increase more rapidly after being once established, than λ 3776. But the separation here, as with the photographic procedure in general, is not as sharp as observed in Parts I and II, and probably for reasons assigned in magnesium. No members of the principal series of doublets were observed. The first one to come into view would be $4p_2$.

Table IV gives a list of the lines observed in the spectrograms. The lines observed belong to the sharp series, *s*, and diffuse series, *d*. The quantum values of the lines are shown by *n* and the sequence in the members by *a*. There are 16 other lines or incipient bandheads, none of which identify with known arc or spark lines. If

[1] This thallium green line λ 5350 in the small spectroscope seemed to occupy about the same position as a line designated as cadmium λ 5378 Part II, page 249 ff. This line in cadmium had been taken for granted from the descriptions in spectroscopic tables. Measurements showed that the line in both cases was λ 5350 and that the line λ 3779 in cadmium was also an impurity of thallium and was the line λ 3776.

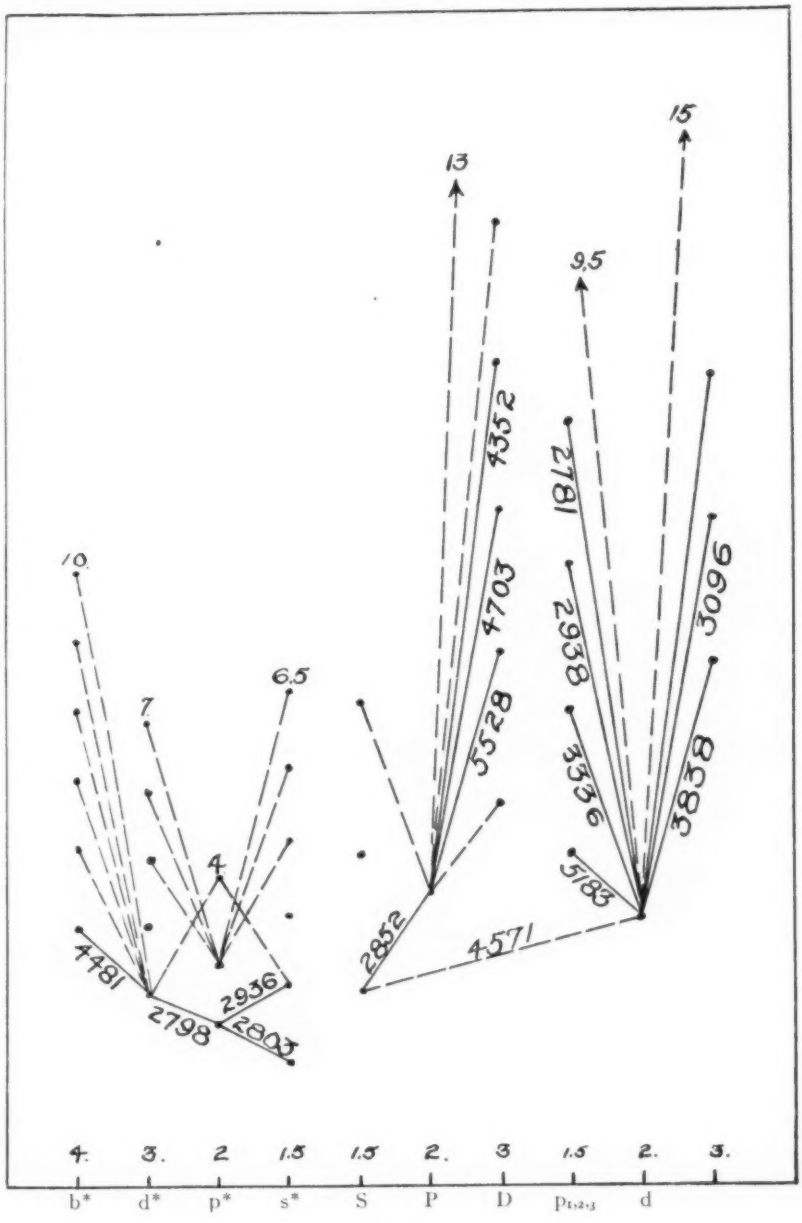

15
13
9.5
10
7
6.5
4
4352
4703
5528
2781
2938
3336
5183
3096
3838
4481
2936
2798
2803
2852
4571
4. 3. 2 1.5 1.5 2. 3 1.5 2. 3.
b* d* p* s* S P D p1,2,3 d

FIG. 3

the intensity of λ 5350 in the visible spectrum were comparable, these lines show an intensity difference of over three thousand fold. All these lines did not appear at the lowest current. For example, when λ 3776 is overexposed one thousand fold at .04 ampere the negative pole line 5.5*s* has not appeared. Let us assume that the rate of development is proportional to the 3.3 power of the current, then at .16 ampere an exposure ten times as long as the first exposure of λ 3776 is equivalent to the one thousand fold overexposure. When such an exposure is made at .16 ampere both 5.5*s* and 1.5*s*

TABLE IV

THALLIUM

Sharp Series, *s*. Doublets			Intensity	Diffuse Series, *d*			Intensity
λλ	*n*	*a*		λλ	*n*	*a*	
5350	1	1.....	.....	3529	3	1′.....	400
3776	1	2.....	3000	3519	3	1.....	400
3230	2	1.....	600	2768	3	2.....	150
2580	2	2.....	50	2921	4	1′.....	20
2826	3	1.....	25	2918	4	1.....	20
2316	3	2.....	1	2380	4	2.....	1—
2666	4	1.....	15	2711	5	1′.....	3
....	4	2.....	.....	2709	5	1.....	3
2585	5	1.....	2	....	5	2.....	.....
....	5	2.....	.....	2609	6	1′.....	
2538	6	1.....	1		6	1.....	1

appear. Hence, even if these lines were not absent at the lower current they have at least developed more rapidly than the 3.3 power of the current. The continuation of this line of analysis shows that λ 3776 is developing less rapidly than the third power of the current. If we compare the later with the earlier current we find that the line must be similar to λ 5461 of mercury, which exterpolated would give a large intercept upon the ordinate axis at zero current. The ratio of pole to middle arc is low for the lines $2.5s_2$, $3.5s_1$, $3d'_1$, $3d_1$, and $3d_2$. Unfortunately the record in the middle of the arc is not complete. For λ 5350 in the visible spectrum at .32 ampere the ratio of pole to middle is less than five.

MAGNESIUM

A brief discussion of this substance has already been given.[1] It was therein recorded that the resonance line λ 4571, Type $1.5S-2p_2$, is a result of "spraying and flaming" process, and that it does not belong in the arc process of the self-sustaining arc, while λ 2852, Type $1.5S-2P$ was found to be the leading line. It was not obtained entirely alone even in the middle of the arc. A group of companion lines seemed to accompany it upon the violet side. To make more than a guess at the identity of these lines, it was necessary to make use of a larger dispersion quartz spectrograph. The University of Minnesota kindly loaned an instrument which made possible the completion of the analyses herein recorded.

The previous study of magnesium was made with salts of magnesium upon carbon. The substitution of the metal in air gave nothing different, the substance rapidly oxidized and flaming was liable to ensue. In the present experiments the arc of the metal was studied in nitrogen and hydrogen under various pressures and in mixtures of these gases. This method did not make it possible to reduce the arc to the point where the leading lines, hereafter mentioned, did not appear along with the resonance line λ 2852. Whether even the middle of the arc was free from the last trace of these lines at the lowest current seemed doubtful.

Table V gives a record of these lines at .16 ampere. To bring out comparative densities, the exposures in the middle of the arc were made twenty and sixty times as long in nitrogen and hydrogen, respectively. Photographs were taken down to .03 ampere where the variations were larger, due to the irregularity in the position of the minute negative pole. Here, too, the continuous and band spectra are troublesome in the long exposures. In the middle of the arc the changes are more pronounced. Lastly there is given in Table V, for the purpose of comparison, a partial list of observations taken by Foote, Meggers, and Mohler for three characteristic potentials in the "separately excited" vacuum arc.[2] To be complete,

[1] *Astrophysical Journal*, **54**, 215, 1921. In the eleventh line from the bottom of page 215, the expression "this line" refers to λ 4571 not to λ 5183.

[2] *Philosophical Magazine*, **42**, 1006, 1921.

there should be a polar column for hydrogen also. The effect of the substitution of hydrogen for nitrogen is to increase the intensities of all lines at the poles, which is just the opposite of the effect of the substitution in the middle of the arc. Also, the lines which

TABLE V

λλ	Intensity			Intensity by F. M., and M.			Type
	Negative Pole	Middle ×20	Middle ×60	20V	10V	7V	
4571	0	.....	.....	20	15	5	$1S-2p_2$
2852	200	150	120	50	50	10	$2P$
5183	160	70	50	7	8	1—	$1.5s_1$
72 67	80	50	30	10	10	1—	$1.5s_2$ $1.5s_3$
2803	60	30	8	10	8	.....	$2p^*_2$
2795	50	25	6	10	9	.....	$2p^*_1$
3838	30	25	12	12	10	.....	$3d_1$
32 29	25	20	10	12	10	.....	$3d_2$ $3d_3$
2798	20	4	1	4	1	.....	$3d^*_2$
90	30	5	1	6	1	.....	$3d^*_1$
2936	25	4	1	6	5	.....	$2.5s^*_2$
29	20	3	1	6	1	.....	$2.5s^*_1$
4481	15	0	0	8	.....	.....	$4b^*$
3097	12	6	2	9	9	.....	$4d_1$
92	.....	.....	.....	6	7	.....	$4d_2$
91	1	5	2	10	9	.....	$4d_3$
3337	12	3	1	10	10	1	$2.5s_1$
32 30	10	3	1	8	8	.....	$2.5s_2$ $2.5s_3$
2779	10	4	2	4	4	.....	.....
83	8	3	1	5	5	.....	.....
81	6	3	1	4	4	.....	$4.5s$
78	6	3	1	4	4	.....	$4.5s$
77	4	2	1	5	5	.....	$4.5s$
5528	4	4	2	1	0	.....	$4D$
4703	2	2	2	15	10	.....	$5D$
2942	2	1–3	0	10	9	.....	$3.5s_1$
38 37	2	2	0	4	5	.....	$3.5s_2$ $3.5s_3$
4351	1	1	1	40	30	.....	$6D$

decrease the most in the middle of the arc, are the ones which increase the most at the poles. For example, at the negative pole in hydrogen the intensities of the lines λ 2803, λ 2798, λ 2936,and λ 4481 which are first terms of four series, theoretically belonging to the excitation of the positive nucleus are about in the ratio 5, 4, 4, 3; whereas, as will be seen from Table V, their respective ratios in nitrogen are about 12, 4, 5, 3. This means an "enhancement"

of the three later lines in hydrogen with respect to λ 2802. Visually the "enhancement" of λ 4481 seems nearer tenfold than 2.4 fold, which would be no contradiction if there is an enhancement in λ 2802 of fourfold. However, it did not appear that the line λ 2802 was more than doubled for a common current upon passing from nitrogen to hydrogen. Although the polar spectrum in hydrogen is stronger than in the nitrogen, there are no extra lines brought out until a current of .22 ampere is reached, when the doublet λ 3104, type $5b^*$ appears. The same doublet has not yet appeared in a nitrogen atmosphere at .32 ampere. (Owing to the heat developed in hydrogen, the form of the apparatus would not permit the use of more than .22 ampere.)

In Table V the negative pole lines are given in the order of the relative intensities of the respective types. It is seen at a glance that the order is quite different in the vacuum arc data by Foote, Meggers, and Mohler. This is all brought about by reason of the fact that the intensities in the self-sustaining arc drop off very rapidly as one passes from the inner toward the outer Bohr rings, whereas by virtue of the freedom of motion in the vacuum arc there is less restraint upon the outer rings. This may be made clearer by the graph, Figure 3. The following brief explanation of this graph is adequate. Taking the equation for the limiting frequency in the Bohr-Sommerfeld theory,

$$\frac{N}{(1+k)^2}=K, \qquad \frac{N}{(1.5+s)^2}=A. \qquad \frac{N}{(2+p)^2}=a',$$

etc., the values

$$1.5+s, \qquad 2+p, \qquad 3+d,$$

etc., are computed. These values are then plotted as ordinates for arbitrary lateral displacements of s, p, d, etc. Then the value unity, or one quantum, is added in succession to represent the successive orbital positions. For lines which represent a recovery from the second ionization, we have for limiting frequency[1]

$$\frac{4N}{(1.5+s^*)^2}=\frac{N}{\left(\frac{1.5+s^*}{2}\right)^2}=A^*.$$

[1] Sommerfeld, *Atombau und Spectrallinien* (3d ed.), pp. 273–276. 457.

Here the value $\frac{1.5+s^*}{2}$ is computed. When one quantum is added to the numerator of this value it increases the graph step by one-half as much as in the case of the first ionization. The lines appear twice as frequently. The square of the distance from any position to the axis of abscissae is proportional to the radius of the electron orbit for that position (this is rigid only for the inner *circular* orbits). A conspicuous difference in the two types of arcs is the outer extension of their respective ring systems. The spectral lines are represented by diagonals running downward from an upper to a lower ring.[1] The types of the lines (see last column, Table V) are designated by the upper limit only, unless this designation would introduce confusion, as in case the line belongs to mixed type.[2] The full diagonal lines represent the types observed in these experiments. The dotted lines correspond to lines observed by Foote, Meggers, and Mohler. The numbers at the tops of the columns represent the quantum limits observed by the latter observers. The outer limits in the two types of arcs is very striking. The contrast is even larger when it is considered that the earlier types of lines in the self-sustaining arc are stronger than in the "separately excited vacuum arc." An effect of pressure or difference in the separation of the molecules of the gaseous medium suggests itself at once as a restraining force. But if this were a sufficient explanation, the ring limits in the second ionization lines would be just as extended as they are in the first ionization process. The other factor which enters in the "field state" is the temperature. The first duty of a self-sustaining arc light is to establish a working temperature; and very evidently, from what has proceeded, the temperature variations in the low current arc are very great. However, even the polar tip is only hot enough to excite the first rings of the second ionization type. It is seen that these rings are essentially no farther from the atom than the first rings of the first ionization type. It is also doubtful whether

[1] Sommerfeld, *ibid.*, p. 432.

[2] Absorption lines would be designated by the lines running in the inverse order. The upper position, a condition of ionization, is the field state before the beginning of radiation. The lower position is the state of the field such that the absorption spectra may be produced. It represents the neutral atom for the first ionization lines, and a simple ionization atom for the double ionization group.

the temperature is ever so low in the self-sustaining arc, that both electrons in this and other bivalent metals do not become excited at the polar tip. In the middle of the arc, particularly with hydrogen gas, we approach a condition which is free from the second ionization group of lines. We see this readily if we are free to assign a resonance to the second group lines λ 2803, λ 2796, and allow a small reflection in the middle of arc of polarized light coming from the poles. We also see that in the middle of the arc the first ionization lines are reduced to the limits of the first ring types, *p*, *d*, *s*. The *2p* triplet is just as persistent as in zinc and cadmium, and the third type more so. In the middle only, then, is there a true separation of types, while at the poles there is no trace of evidence that we may build up a *one-ionization* spectrum, and then advance to a hotter state and form a *second ionization* spectrum, or that we might designate the former an "*arc spectrum*" and the latter a "*spark spectrum.*"[1]

The real difference between polar and middle illumination is greater than a spectrogram will show, because the polar illumination wanders and does not stay upon the slit but part of the time. The envelope which surrounds the polar light only falls upon the slit, and this envelope has the same character as the middle of the arc. In the visible spectrum, where one may follow the lines with the eye, one may obtain a more faithful record by making observations only at the instant the desired polar point is upon the slit. Accordingly, attention was directed to two lines in the visible spectrum which have been the subject of much contention, and which have varying ratios in different types of stars. These lines are λ 4481, Type $3d^*-4b^*$, and line λ 4352, Type $2P-6D$. In some stars λ 4481 is the stronger, in others the weaker line. One theory has been that λ 4481 is due to a "hotter" star. This opinion is based upon the "enhancement" of the line upon passing from

[1] It is not advantageous to call things by a name, and tacitly admit that the name means something else. Sommerfeld, *op. cit.*, p. 457.

Evidence was given in Part II that polar illumination grows hotter as current increases, and that the later excitations therein were later ring formations of types already well defined. The "Zone" concept of type formation introduced by Lenard (*Annalen der Physik*, **11**, 636, 1903), has some features in common with the present results but its discussion would lead too far.

the arc to the spark discharge. The following examination confirms this opinion. Because of the low visibility of λ 4352, the writer has used the next earlier line λ 4703, Type $2P-5D$. Since λ 4481 does not appear in the arc in air,[1] it was necessary to use a nitrogen atmosphere. For both lines the current was varied progressively from .04 ampere to .32 ampere. (The latter only could be used for very brief intervals due to the heating of the apparatus.) The intensities of the lines were compared through ray filters. As mentioned in Parts I and II, there is a large error in this procedure, but one can get a general perspective of the changes, at least. The growth of polar lines is indicated in equation 2. The exponent x lies between 3 and 3.3. Analysis shows that the value of k, which represents the intrinsic brightness coefficient, is at least eight times as large for λ 4481 as for λ 4703 at the negative pole. Next let us take the middle of the arc and compare the lines. Upon increasing the current from .04 ampere to .12 ampere, the line λ 4703 has about the same intensity in the middle of the arc as it had at .04 ampere at the pole; and three cubed is twenty-seven, which compares very favorably with the ratio of pole to middle, twenty-five, found in Table V. At .32 ampere the middle intensity of the line λ 4481 has not reached its polar intensity at .04 ampere. While at .24 ampere, line λ 4481 at the middle of the arc was traceable rather transiently and, even then, decidedly weaker than λ 4307 in the middle of the arc at .12 ampere. From this we see that, in the middle of the arc, the order of the intensities of the lines is just the reverse of that at the negative pole. Following Parts I and II we may characterize λ 4481 as a rigidly short line, and λ 4703 as an easily long one. This difference is explained by saying λ 4703 arises from a first, and λ 4481 from a second, ionization. The old theory that λ 4481 represents a *hotter* state is equally valid, for this is required, when energy magnitudes are considered in the ionization process. The observations make it clear also why one may obtain such anomalies as the experiments by Hartmann,[2] wherein it was found that the line λ 4481 increased in intensity as

[1] Some observers record this line in the air arc of the metals. The writer has only seen a trace of it with clean electrodes and never with an oxidized surface.

[2] Hartmann and Eberhart, *Astrophysical Journal*, **17**, 229, 1903.

the current decreased. We have noted above that the line increases at least with the cube of the current, and that it is *initially* absent at the middle of the arc. Hartmann operated with a low voltage arc which is necessarily a short one even with large currents, and which for decreasing currents becomes progressively shorter. For the lowest current Hartmann could not sustain the arc, and in order to secure a photogram he had to strike the arc continuously for a long period. In the light of the foregoing experiments, for the stronger currents Hartmann photographed the middle of the arc, then as the current and arc length decreased, he progressively received more polar illumination, and for his weakest current his spectrogram was entirely a polar view of the line.

It is plain, from the foregoing considerations, that lines develop at different rates with change in current, and that there are differences in polar and middle illuminations; and that the "enhancement" of some lines is quite marked in these changes. We see evidence for stages in the development represented, first by the line λ 2852, followed by the ionization types *s*, *p*, *d;* and the latter closely followed a second resonance type of lines $\lambda\lambda$ 2803, 2796, which are the basic lines of another ionization.[1] Finally, these are closely followed by lines which represent a complete second ionization. The whole is a condition of affairs which overlap at the poles of the arc but which is brought out by comparing the middle arc under different conditions. At the poles, in fact, when the current is carried down to the very limit at which the arc will operate, there are lines which theoretically have been assigned to both a first and a second stage of ionization.[2] The writer has called attention to the discontinuity in the current when the arc changes from the glow to the true arc discharge.[3] This discontinuity has its origin in the change of polar "drop" which occurs upon the establishment of an ionization at the electrodes. If there be a later stage or two stages in the ionization it is not observable. Since lines, which have been designated as representing two sequences in the loss of electrons for bivalent elements, are traceable to the very lowest limits in cur-

[1] Sommerfeld, *op. cit.*, p. 456.

[2] Sommerfeld, *ibid.*

[3] Chrisler, *Astrophysical Journal*, **54**, 273, 1921.

rent for the self-sustaining arc, it is fair to conclude that the establishment of the arc always develops enough heat (or liberates enough energy) to free *both* valence electrons.

As pointed out above, long-sustained photographic exposure upon a shifting polar tip complicates the results. In the visible spectrum one can make observations at the instant the image of the pole is upon the slit of the spectroscope. To accomplish this, the writer took up again barium, whose P_1 line λ 5536, and $2p_2$*, λ 4554, and $2p_1$*, λ 4934 are in the visible spectrum. The arc was now operated in nitrogen and hydrogen under varying pressures. As the pressure falls, the current under which the true arc will operate increases, but at lower pressure the change in the current is larger when the form of the arc changes, and the changes are more frequent. These changes occur only when one is operating near the contact of the glow and true arc characteristics.[1] The observations confirm the earlier record that the last appearance of the barium line λ 5536 is in a broad band emitted by a glowing particle which was cooling below the limit of current for the true arc, and that $2p_2$* always appeared at the pole at the instant of the leap from glow to arc form of discharge. The conclusion that λ 5900 and λ 4525, Types 2.5*s**, are later in their formation was also reconfirmed. This latter type corresponds to λ 2936 and λ 2938 of magnesium and they are strictly short like λ 4481 in magnesium. Hence it appears that one may designate "extreme shortness" in lines as a characteristic of second ionization; and that the long features of the fundamental lines of the excited positive nucleus, represented in barium by λ 4554 and λ 4934, and in magnesium by λ 2803 and λ 2795, arise from a resonance of a positive nucleus accompanying the ionization spectrum of the atom.

I have called attention to the "enhancement" of the cadmium doublet λ 5379, λ 5339 and to the zinc doublet λ 4924, λ 4912, which do not appear in the air arc—a feature which, in so far as the present observations go, they have in common with the magnesium doublet λ 4481. Since the latter shows very conspicuous enhancement in various stars, and is the most conspicuously enhanced line in the magnesium arc, it was desirable to know whether the cadmium and

[1] Chrisler, *loc. cit.*

zinc doublets above designated would appear in a hydrogen or a nitrogen atmosphere. They were not found, and probably represent a "hotter" stage of excitation than λ 4481. Hartmann obtained all of these lines in an arc under water, which he called similar to the arc in hydrogen, and drew the conclusion that they are "cold" lines and under a subtle influence of hydrogen. This conclusion appealed to Kayser,[1] who says "dass hier von einer erheblichen Steigerung der Temperatur nicht gesprochen werden kann, dass alle Schlüsse auf Temperatur aus dem Auftreten der Linien verkehrt sind." But it is this conclusion which is as completely "twisted about" (*verkehrt*), as the arc is "*under*" not "*through*" water. The energy consumption of the arc is large, the pole drop is large and restricted, and therefore the polar illumination represents a *very* hot condition.

BRACE LABORATORY
UNIVERSITY OF NEBRASKA
March 1923

[1] Cf. *Handbuch der Spectroscopie*, **5**, 710.

EXCITATION STAGES IN THE OPEN ARC-LIGHT SPECTRA

PART IV. HYDROGEN, AIR, WATER-VAPOR, PRESSURE EFFECT, MIXED ELECTRODES

By B. E. MOORE

ABSTRACT

The effect of the medium (hydrogen, air, water-vapor) upon the spectrum.—If hydrogen is substituted for air or nitrogen the middle of the arc is cooler, but it is hotter at the electrodes because of the increased polar drop of potential; and there is a corresponding effect upon the spectra of the electrode materials. The many-lined spectrum of hydrogen is more easily excited than the Balmer series. These lines have two features in common with air bands, nitrogen bands, certain chloride bands, and a large group of water-vapor lines, all of which were examined, viz., they are all easily excited, and they form only on the outside of the hot polar discharge. The inference is that the products which produce these types are dissociated by high heat.

The effect upon the spectrum of pressure and of mixed electrodes.—Decreased pressure is accompanied by a decrease in temperature and results in less contrast in the series lines. The spectral lines appear first at the negative pole, even when the substance in question is located at the positive pole. A low-volatilizing substance at the positive pole increases the intensity of the spectrum in the middle of the arc of a higher volatilizing substance used as a negative electrode. This is attributed to the vapor of the positive pole which conserves the heat in the middle of the arc and raises the temperature. As the vaporized electrode material increases in the arc it progressively suppresses the hydrogen spectrum. The effect begins at a lower current with an easily volatilized substance, e.g., cadmium, than it does with a substance which volatilizes at a higher temperature, such as copper.

General conclusion.—The changes due to pressure and temperature together with the variations in electron velocity are sufficient to explain all of the differences which have been noted in the spectra.

HYDROGEN

Attention was first directed to the effect of the substitution of hydrogen for heavier gases upon the spectra of the metallic electrodes. Naturally the spectra of hydrogen, in so far as they were present, could be noted also.

THE GENERAL EFFECT OF HYDROGEN

For all electrode materials tested, the substitution of hydrogen for air or nitrogen reduced the intensity of the spectra of the metals in the middle of the arc. At the poles, the effect was not always uniform. There was always some strengthening of the lines upon passing to a hydrogen atmosphere. The change ought to be considered in the light of the electrical behavior of the arc in hydrogen. The potential fall across the arc is greatly increased when hydrogen

is substituted for air. The writer will make use of some unpublished observations by Mr. V. L. Chrisler in this laboratory, who finds that the principal part of this important change takes place at the electrodes. This gives both greater electron velocity and greater heating at the electrodes. The middle of the arc upon the other hand is not so hot. This is not because less energy is produced there, but because hydrogen rapidly conducts the heat away from the arc to the inclosing vessel.[1] The greater velocity at the electrodes would bespeak a more extended pole effect as well as a more intense polar action. The former is not realized. The simple reason seems to be that the cooler middle encroaches upon the poles more than in the heavier gases. So that in many cases there is an actual shrinking in the length of the polar illumination, while, it, at the same time, becomes brighter. There is not much difference, for a given current, in the actual number of lines realized whether air, nitrogen, or hydrogen is used, i.e., the higher potential is counteracted by the natural disposition of hydrogen to carry the heat away from the arc. About the largest difference which can be realized has already been indicated under magnesium, Part III.

Most of the recent investigations upon spectral emissions have looked upon the lines as the result of electronic impact, wherein some definite potential is required to impart a velocity to the electrons. These investigations have been disposed to overlook the "heat factor" or "kinetic impacts" in the functioning of the arc. The action of the latter kind of motion is to sustain a field of vibration, while the special lines arise from a particular type of vibration in a particular form of unit in this field. As a definite energy impact is required to produce a definite radiation, one may resolve the energy into two components, viz.: the translational component due to the current flow, and the "heat factor" component. Naturally, in the self-sustaining arc, we may then regard the "heat factor" as the sum of all translational impacts in the current circuit which fall below the magnitude required to produce radiation excitations. Since most of this energy passes from the arc by convection and from the electrodes by conduction, and since this

[1] Stark, *Annalen der Physik*, **18**, 213, 1915, finds that heavier gases conserve more heat and raise the temperature at constant pressure and constant current.

energy in the field must be kept at a certain definite value to prevent the arc from going out, it follows that a greater part of the energy of the arc is spent in maintaining this field energy. Hence the greatest field energy, or highest temperature, lies in close proximity to the electrodes, while the field energy in the middle of the arc depends upon the conserving power of the gases and vapors which may be present.

This conserving power is the least for hydrogen of all gases, and therefore the contrast between polar and middle illumination is the greatest for this gas. As the current increases the electrodes volatilize progressively, and for very large currents one approaches a condition where the middle of the arc is a vapor of the electrode material. These vapors are heavier than the surrounding medium, they conserve more heat,[1] the temperature rapidly rises, and the spectrum approaches its polar character.

We shall see that the results conform to the foregoing outline of the effect of the medium upon the spectra. There were two spectra noted, viz.: the Balmer series and the many-lined spectrum which is frequently designated as the "second spectrum." Three different electrodes were used, cadmium, zinc, and copper. The lines are weakest with cadmium and strongest with copper electrodes. This difference becomes distinct as the current increases. In harmony with the foregoing presentation, the explanation would be that cadmium volatilizes before copper, and begins to substitute its vapor in the middle of the arc for hydrogen at a lower current than copper. Four lines of the Balmer series were observed, but attention was directed almost entirely to the red line, *Hα*. The description will be given for the cadmium electrodes and comparisons given with the cadmium red line λ 6438, Type $2P-3d$.[2] We have, in the neighborhood of this line, the Hα line and some important "second spectrum" lines. We can best understand the condition by trying to visualize them by means of Figure 1.

We will start with an extreme contrast case, about .1 ampere to .12 ampere, when the second spectrum lines have the appearance *a*,

[1] Stark, *ibid.*

[2] In Part II numerical values represented *sequences*, here they represent *quanta*, which is the customary notation.

the cadmium line the polar appearance *b*, and Hα the appearance *c*. Now introduce a 1 per cent transmission filter and the lines appear as shown by *a'*, *b'*, *c'*, respectively. If the current is reduced, *a* takes the form *a'* and *c* the type *c'*, but *b* disappears. The line *a* persists the longest, although *c* has the advantage of a polar formation. If one increases the current, *b* and *c* increase relatively and then absolutely as to *a*. Ultimately *b* passes *c* in brightness and becomes a long line. By this time the *a* line has nearly disappeared. When the cadmium line reaches this stage there begins a gradual absolute diminution of the Hα line. A similar weakening of band lines in the polar region was recorded in Part I, Figure 2, page 206. The explanation is the same. This type of radiation does not take place in the polar illumination, but only in the envelope surrounding it, whereas the Hα line is excited in the hot polar discharge. This indicates that hydrogen is completely dissociated into atoms within the polar illumination and that the *a* type of radiation is a molecular disturbance. We must conclude then that the "*second* hydrogen spectrum" is the *first* hydrogen excitation—a conclusion which may not be generally accepted. The line λ 6438 was found in Part II to belong to Stage III, which, from its Type $2P-3D$, is a rather tardy appearance. The line is an early polar type, but it is not of the rigid polar type represented by λ 4481 magnesium, Type $3d^*-4b^*$. The question then arises: Is it a later appearing type than Hα as appears at first glance from Figure 1? The following consideration makes this doubtful. The Hα radiation begins when the middle of the arc is almost pure hydrogen. The cadmium red line is not strong until very appreciable volatilization of the electrodes begins. When the heat is large enough to carry a considerable amount of the material into the arc the line surpasses Hα in brightness. This is further confirmed by the action of copper. Above .2 ampere with copper electrodes the Hα line is much brighter than when cadmium is used. With the chamber, which has been used, larger currents could be obtained only for brief intervals, but rater transient observations at .25 ampere point to a further

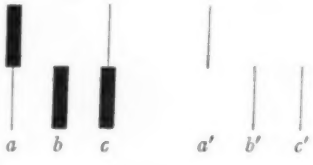


FIG. 1

increase in Hα with copper electrodes, and to an absolute diminution of the same line with cadmium electrodes. The writer is rather forced to the conclusion that if one is given temperatures high enough to insure the presence of both hydrogen and cadmium in the gaseous state, this third-stage cadmium line is essentially more easily generated than the Hα line.

AIR SPECTRA

This subject has already been noticed under band spectra, Part I, page 206; Figure 2 therein used doubtless refers to the character of a chloride band. The writer has not felt that the dispersion at hand afforded him a sufficiently accurate means of identifying these bands by their wave-lengths. Their common occurrence for different electrode materials leaves no doubt as to their air origin. Instead of approaching the polar behavior of these types as indicated by the foregoing figure, one may use the photographic method. It may be recalled that middle arc lines require from ten to a thousand times as long an exposure as the same lines at the negative pole. These bands cropped out more in the middle of the arc. The question arose whether they would be as strong at the poles as at the middle of the arc, if the exposure at the pole were continued as long as at the middle. A test of this kind showed that the lines are absolutely much weaker in the pole illumination, and all the effect actually observed might be attributed to a cooler envelope surrounding the polar glow. There are some bands which have their origin at the pole face; these are very strong in the glow arc of copper, and they nearly disappear upon change to the true arc.

Nitrogen contains nothing unnoticed in air.

WATER VAPOR

In Part II, page 217, two lines λλ 3069, 3064 are recorded which persisted under all conditions. Professor A. Fowler has kindly suggested that these lines had their origin in water-vapor. This conclusion seemed very acceptable, but it was desirable to make some test of the matter. An arc light of copper was tried in atmosphere of air, nitrogen, and hydrogen. The gases were passed over phosphorus pentoxide and a vessel of pentoxide was placed in the

arc-light chamber. The lines were still traceable. Thinking that this process had possibly not removed the last trace of moisture, a vessel of hot water was placed in the chamber filled with nitrogen; there was now a profuse supply of banded lines in this region.[1] The electrodes were magnesium and this water-vapor did not seem to interfere with the polar observations upon it. An overexposure for these lines at the pole in the saturated water-vapor was not made but the lines appear in the middle of the arc in the foregoing attempt to analyze the bands, and they did not come out there upon overexposure at the pole. Hence it must be that water-vapor dissociates in the position of the polar illumination. This feature it has in common with air bands, nitrogen bands, some of the chloride bands, some metallic oxides, and the hydrogen "second" spectrum.

The so-called cyanogen band head at λ 3883 was not very distinct except with carbon electrodes, and no definite results can be stated for it.

THE EFFECTS OF PRESSURE

It has been pointed out in Part II that the Bohr theory anticipates more rings, and therefore more lines, in an attenuated gas.[2] In Part III, Table V, one reason assigned for difference in the relative intensities of the writer's lines and those of Foote, Meggers, and Mohler was the difference in the attenuation of the gases.

In an attempt to reduce the pressure in the self-sustaining arc, one is confronted with the fact that the discharge passes from the arc to the glow at a larger current when the pressure is reduced. With a current of .1 ampere it was difficult to sustain the arc with a pressure of .1 atmosphere. Therefore, the pressures used for comparison were about $\frac{1}{8}$ and 1 atmosphere. The current was varied from .1 ampere to .32 ampere. Zinc and cadmium electrodes in nitrogen were used. An effort was made to extend the principal series triplet, p_i. This resulted in disappointment. The same number of lines appeared in both cases. But something else was apparently happening. The leading member, $2p_i$ seemed brighter under the higher pressure. Hence some attention was directed to $2p_i$, cadmium, and the arc subjected to pressures .1, 1, and 10

[1] Sommerfeld, *op. cit.*, p. 527.

[2] Part II, p. 258, *Philosophical Magazine*, **26**, 9, 1913.

atmospheres. An assistant adjusted the current at values unknown to the observer, who examined the lines through filters. The readings were taken from .1 ampere to .25 ampere. The most uniform results were obtained at .19–.21 ampere. Here the ratio of intensity at 1 to that at $\frac{1}{10}$ atmosphere was $(6.5 \pm 2) : 1$. The ratio for 10 to 1 atmosphere was $(4 \pm 1.2) : 1$, where 1.2 is the average deviation from the mean. At lower pressure, then, there is less relative variation in the sequence of series terms. Evidently, also, one must reckon with the temperature differences when contemplating the differences in lines.[1] An increase in temperature and a decrease in pressure is the complete condition for obtaining a greater number of lines. In the arc a decreasing pressure gives an opportunity for the field to carry the heat away more rapidly and hence it may happen that a decrease in temperature becomes more important than a decrease in pressure. A great many factors would have to be known to settle such a point. In a strong gravitational field like the sun, one may have large fall (outward from the surface) in pressure and a comparatively small temperature gradient, such a condition might bring about such anomalies as a dominant "colder" inner line and a dominant "hotter" outer or high altitude line. Thus calcium λ 4227, resonance type, nucleus Ca, predominates at low solar levels, while calcium λ 3968 and λ 3933, second electron resonance type $2p_i$, nucleus Ca^+, predominates at higher altitudes, although at constant pressure it would require a higher temperature than the former line.[2]

MIXED ELECTRODES

The potential (and current) at which the change from glow to true arc occurs, increased roughly as the volatility of the electrode material decreases, and the spectral lines, in so far as noted, appear first at the negative electrode.[3] Where then, would the lines of a low boiling-point metal first appear if the substance were used as positive pole and some less volatile substance were used as negative pole? To answer this question, cadmium and zinc were tried as lower positive and copper as upper negative electrode.[4] A glow

[1] Saha, *Philosophical Magazine*, **40**, 478, 809, 1920; **41**, 267, 1921.

[2] Part II, p. 271.

[3] V. L. Chrisler.

[4] To reverse the electrodes does not change the result.

current with copper electrode can be maintained at much larger currents and for a longer time than with cadmium electrodes. With a copper-cadmium combination the copper will in due time become tarnished with the other metal. When the discharge passes more readily to the true arc form, such a tarnishing is sure to take place in time. But this is rather immaterial, as some of the lines of cadmium and zinc may be seen upon the first flare which accompanies the change in form of the arc. These lines appear at the copper instead of at the cadmium or zinc electrode. Next, copper was used as a positive lower electrode and platinum as a negative. Attention was directed to the copper line λ 5105. It appears first at the platinum electrode. Then a photograph of the spectrum was taken at the negative electrode. This reveals the copper-leading doublet and no line which could be attributed to platinum. Of course, tarnish might be effective here.

The atoms had crossed the electron field with no impact strong enough to cause an excitation until they came within the negative pole drop.

An attempt was made to find the copper doublet within the glow discharge. This pair is about the most favorable for this purpose. Copper was used for both electrodes and the chamber filled with nitrogen. This glow discharge may be described as a positive column which terminates in a minute Faraday dark space, near the negative electrode. Over the latter is a violet film, which yields bands. The positive column shows brightest adjacent to this dark space. A long exposure shows the copper doublet very weakly and the bands very brilliantly. The copper lines are lower, which shows that their position was in the head of the positive column and not in the band region. If there had been occasional minute true arc discharges the result would have been the same. So that there is no conclusive evidence that resonance excitation ever occurs in the glow arc.

Next, attention was directed to the effect of low volatile materials upon higher volatile ones. The introduction of materials in this way lowers the potential fall across the arc somewhat, but increases the intensities of lines instead of decreasing them. Let us take as an example the iron arc. At .1 ampere some lines are distinct but

not strong in the negative (upper) discharge. There are no lines beyond this polar region, which is suspended from the electrode like a drop. If we now substitute for the lower positive electrode lead, cadmium, magnesium, or mercury, a threadlike discharge comes from the tip of the positive electrode, and drives into the negative drop, making it heart-shaped upon the bottom. In this threadlike column there may now be found numerous iron lines, although the current is regulated to the same value as before.

This result would follow directly from an increase in temperature due to the conservation of heat upon the substitution of heavier vapors for nitrogen.

Brace Laboratory
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ON THE RADIATION AND TEMPERATURE OF THE EXTERNAL PHOTOSPHERIC LAYERS

By RAGNAR LUNDBLAD

ABSTRACT

Radiation, temperature, and optical properties of the photospheric layers of the sun.—Starting from the observations on the distribution of the energy over the sun's disk, the optical properties of the photospheric layers and the state of radiation within them are examined as closely as possible with a minimum of a priori assumptions. This is accomplished by a new method of inverting the integral equations of the problem. The solution develops two functions which can be said to express the amount of radiation converging toward and diverging from an element of any layer. From these can be computed the *intensity of radiation of each separate wave-length at any depth and in any direction*, and *the conditions for radiative equilibrium;* and in addition the temperature and extinction coefficients of each layer for each wave-length. From these theoretical results it is found that the outermost layers of the sun have an *absolute temperature* of 4500°, and that the temperature increases with the depth. Only a small fraction of the radiation reaches the limit of the atmosphere from the layer at which the temperature is 7500° K, and in this radiation there is a maximum at $\lambda=0.48\ \mu$ at which point 11.5 per cent of the radiation emitted straight outward reaches the outer atmosphere. The *extinction-coefficient* is very large in the extreme ultra-violet. It has a minimum at wave-length $\lambda=0.48\ \mu$ and a maximum at about $\lambda=0.95\ \mu$. A superior limit of the ratio of *scattering* to real absorption is assigned. The scattering is entirely negligible within the infra-red and visible regions.

By the well-known investigations of Schuster,[1] Schwarzschild,[2] and Emden[3] the theory of the transport of radiant energy through an atmosphere has obtained its solid foundation. Several authors have afterward dealt with the same subject, especially in its application on the solar envelope. Some [4] of them assume real absorption and radiative equilibrium, others[5] consider the scattering the most prominent and conclusive feature of the optics of the solar atmosphere. Common to all of them is the course of the investigation,

[1] A. Schuster, *Astrophysical Journal*, **21**, 1, 1905.

[2] K. Schwarzschild, *Nachrichten v.d. Gesellschaft d. Wiss. zu Göttingen, Math.-phys. Kl.*, p. 41, 1906; *Sitz.-berichte d. Preuss. Akad. d. Wiss.*, p. 1183, 1914.

[3] R. Emden, *Sitz.-berichte d. Bayer Akad. d. Wiss., Math.-phys. Kl.*, p. 55, 1913.

[4] E. Öpik, *Astronomische Nachrichten*, **198**, 48, 1914; B. Lindblad, Diss. Upsala, 1920; E. A. Milne, *Monthly Notices*, **81**, 361, 375, 1921.

[5] A. Defant, *Sitz.-berichte d. Akad. d. Wiss. in Wien, Math.-phys. Kl.*, Abt. IIa, **125**, 514, 1916; J. Spijkerboer, *Arch. Neerl.* (IIIA), **5**, 1, 1918; P. H. van Cittert, *Proc. Amsterdam*, **22**, 73, 1919; H. Groot, *Proc. Amsterdam*, **22**, 89, 1919; *Physica*, 1, 7, 49, 1921.

starting with some assumption and then deducing the intensity distribution of the sun's disk; if the constants of the problem are disposed of in a convenient manner, the calculated distribution can be made to suit the observations rather well in several cases.

The disposition of this paper is the converse one. Making no assumptions as to the thermal equilibrium or the amount of scattering and real absorption we proceed from the observations on the intensity distribution over the sun's disk, computing two functions, which are characteristic of the radiative state of the various layers; then the intensity of radiation of each specified wave-length at each specified layer and in each specified direction, the temperature, and the coefficients of extinction of each layer are readily obtained; and it proves possible to assign limits of the coefficients of absorption and scattering separately. Ultimately, we shall see that the conditions of radiative equilibrium are satisfied.

1. THE MATHEMATICAL LAWS OF THE INTENSITY OF RADIATION TRAVELING THROUGH AN ABSORBING, SCATTERING, AND RADIATING ATMOSPHERE

In the sequel we shall assume that the sun is a great sphere, composed of concentric layers of radiating, absorbing, and scattering matter; the temperature of each layer and its coefficients of absorption and scattering for every wave-length λ being functions of the distance r from the center of the solar globe, and of this distance alone.

In the first place, we have to formulate the equation of the intensity change of a beam during its passage through an infinitesimal layer. We specify the direction of the beam by the cosine of the angle ζ between the beam and the outward normal to the layer, the quantity $\xi = \cos\zeta$ evidently being positive or negative according as the beam is running outward or inward. From reasons of symmetry we conclude that the intensity of the radiation of the wave-length λ must be a function of the two variables r and ξ only. Call this function $J_\lambda(r, \xi)$, and let $\alpha_\lambda(r)$ and $\beta_\lambda(r)$ denote the absorption and scattering coefficients of the layer, and $E_\lambda(r)$ the emissive power of an absolutely black body at the temperature prevailing in the layer.

According to well-known theoretical considerations the radiation round a scattering element would be partially polarized and more or less irregularly distributed. If the radiating layers of the sun were constituted like the earth's atmosphere we, of course, should have to pay regard to these results. The entire lack of polarization of the sunlight proves, however, that the analogy does not hold; the scattered radiation seems in fact more likely to be equally, or at least approximately equally, distributed over all directions, if it is of any account at all. Since every element of the photosphere is the convergency point of an infinitude of beams running from all directions and fairly equally distributed over at least one hemisphere, there is at all events no risk in assuming that the scattering works equally in all directions.

Then the energy balance of the beam is given by the equation

$$dJ_\lambda(r, \xi) = -[\alpha_\lambda(r)+\beta_\lambda(r)]J_\lambda(r, \xi)\frac{dr}{\xi}+[\alpha_\lambda(r) \cdot E_\lambda(r) + \beta_\lambda(r) \cdot G_\lambda(r)]\frac{dr}{\xi}, \quad (1)$$

where the "collustrivity" function

$$G_\lambda(r) = \tfrac{1}{2}\int_0^\pi J_\lambda(r, \cos\zeta)\sin\zeta\, d\zeta = \tfrac{1}{2}\int_{-1}^{+1} J_\lambda(r, \xi)d\xi \quad (2)$$

expresses the average intensity of the radiation converging toward an element of the layer from all directions. The left-hand side of the equation (1) signifies the total energy gain of the beam during its transit through the layer; the first term to the right stands for the attenuation by absorption and scattering, and the second term for the increase of intensity by emission and scattering. The equation holds for the radiation inward ($\xi<0$) as well as for the radiation outward ($\xi>0$).

Strictly speaking, the quantity ξ is a function of r, because the angle of incidence against the successive layers slowly varies as the beam traverses distances which cannot be neglected in comparison with the radius of the sun. But since we need not take such

enormous distances into consideration, we are allowed to treat ξ as a constant.

Then the equation (1) can be reduced to a linear differential equation with constant coefficients by means of the variable change

$$m_\lambda = \int_r^R [a_\lambda(r)+\beta_\lambda(r)]dr; \qquad dm_\lambda = -[a_\lambda(r)+\beta_\lambda(r)]dr; \tag{3}$$

where R is the radius of the sun. The new variable m_λ, which is a function of the wave-length, is called the optical mass with regard to the wave-length λ. It is zero at the external limit of the sun's atmosphere and increases inward. J, E, and G have now to be taken as functions of m. In order to avoid too complicated notation we retain the old symbols, e.g., understanding by $J(m, \xi)$ the value of $J(r, \xi)$ at the layer whose centri distance r corresponds to m by (3). The differential equation is found to be

$$\xi\frac{dJ}{dm} - J = -H, \tag{4}$$

where H denotes the function

$$H_\lambda(m) = \frac{aE+\beta G}{a+\beta} \tag{5}$$

which we shall call the "emissivity" function of the layer m. In (4) m is the independent and J the dependent variable; ξ is a constant parameter and H a function of m. H, J, and m are altogether functions of λ.

The two functions G and H now introduced are the very functions ruling the optical behavior of the medium.

On integrating (4) we obtain

$$J(m, \xi) = J(\mu, \xi)e^{\frac{m-\mu}{\xi}} - e^{\frac{m}{\xi}}\int_\mu^m \frac{H(m)}{\xi} e^{-\frac{m}{\xi}}dm, \tag{6}$$

where μ is an arbitrary constant.

According to the direction of the beam we dispose of μ in a different way. If $\xi>0$ (outward radiation), we put $\mu=\infty$ and obtain the equation

$$J(m,\xi)_{\xi>0} = e^{\frac{m}{\xi}} \int_m^\infty \frac{H(m)}{\xi} e^{-\frac{m}{\xi}} dm ; \tag{7}$$

if $\xi<0$ (inward radiation), we put $\mu=0$ and obtain (because $J[0, \xi]=0$ for this radiation):

$$J(m,\xi)_{\xi<0} = -e^{\frac{m}{\xi}} \int_0^m \frac{H(m)}{\xi} e^{-\frac{m}{\xi}} dm . \tag{8}$$

These are the fundamental laws of the radiation in the photospheric layers.

2. THE SOLUTION OF THE INTEGRAL EQUATIONS

The function $J(0, \xi)$ may be determined experimentally by measuring the intensity distribution over the sun's disk. The problem which we are now going to solve, to begin with from a purely theoretical point of view, is to determine the functions $H(m)$ and $J(m, \xi)$ for any values of m and ξ, the function $J(0, \xi)$ being known.

The solution is most easily achieved in an indirect way, starting from the converse problem to compute $J(0, \xi)$ when $H(m)$ is known. We assume the function $H(m)$ to be an empirical function, tabulated or graphically represented, but unknown as to its analytical expression. With a design to found the necessary computations on a simple algebraical expression, we begin with approximating $H(m)$ as a polynomial of the Nth degree

$$H(m) = \sum_{i=0}^{N} a_i m^i . \tag{9}$$

From the physical sense of $H(m)$ it follows that $H(m)$ is a continuous function of m with a restricted number of maxima and minima. The approximation will, therefore, probably prove very good even for a relatively small N and will naturally grow better the higher we

choose N. For the theoretical part we need not specify N at all.

Then, substituting (9) into (7) and (8) we obtain

$$J(m,\xi)_{\xi>0} = e^{\frac{m}{\xi}} \sum_{i=0}^{N} a_i \int_m^\infty \frac{m^i e^{-\frac{m}{\xi}}}{\xi} dm = \sum_{i=0}^{N} \frac{m^i}{i!} \sum_{k=0}^{N-i} (i+k)!\, a_{i+k} \xi^k ; \quad (10)$$

$$J(m,\xi)_{\xi<0} = -e^{\frac{m}{\xi}} \sum_{i=0}^{N} a_i \int_0^m \frac{m^i e^{-\frac{m}{\xi}}}{\xi} dm = \sum_{i=0}^{N} \frac{m^i}{i!} \sum_{k=0}^{N-i} (i+k)!\, a_{i+k} \xi^k$$

$$- e^{\frac{m}{\xi}} \sum_{k=0}^{N} k!\, a_k \xi^k ; \quad (11)$$

where $0!$ is to denote 1; the terms are arranged according to ascending powers of m.

From these equations the radiation at the external limit of the solar atmosphere is found by putting $m=0$:

$$J(0,\xi)_{\xi>0} = \sum_{k=0}^{N} k!\, a_k \xi^k ; \quad (12)$$

$$J(0,\xi)_{\xi<0} = 0. \quad (13)$$

The latter equation expresses only that there is no radiation toward the sun from the surrounding universe. The function (12) is the more interesting because of its peculiar analytical form.

In fact, if the functions $H(m)$ and $J(0,\xi)$ are expanded in series of m and ξ, respectively:

$$H(m) = \sum_{i=0}^{N} a_i m^i ; \quad (9)$$

$$J(0,\xi) = \sum_{i=0}^{N} b_i \xi^i ; \quad (14)$$

the following simple relation holds between the coefficients a_i and b_i:

$$a_i = \frac{b_i}{i!}. \tag{15}$$

This connection is the essential basis of the method proposed in this paper. It obviously implies the solution of our original problem.

For, having observed the intensity distribution over the sun's disk, we know the function $J(0, \xi)$ empirically. The values observed can always be represented by means of an expansion like (14). According to a theorem due to Weierstrass we are able to reduce the difference between the function J and the polynomial (14) below any limit given in advance by choosing N great enough. Having found such a polynomial, we obtain by (15):

$$H(m) = \sum_{i=0}^{N} \frac{b_i}{i!} m^i ; \tag{16}$$

and according to (10) and (11)

$$J(m, \xi)_{\xi>0} = \sum_{i=0}^{N} \frac{m^i}{i!} \sum_{k=0}^{N-i} b_{i+k} \xi^k ; \tag{17}$$

$$J(m, \xi)_{\xi<0} = \sum_{i=0}^{N} \frac{m^i}{i!} \sum_{k=0}^{N-i} b_{i+k} \xi^k - e^{\frac{m}{\xi}} \sum_{k=0}^{N} b_k \xi^k . \tag{18}$$

Finally the function $G(m)$ can be calculated from (2), (17), and (18):

$$G(m) = \tfrac{1}{2} \int_{-1}^{+1} \sum_{i=0}^{N} \frac{m^i}{i!} \sum_{k=0}^{N-i} b_{i+k} \xi^k d\xi - \tfrac{1}{2} \int_{-1}^{0} e^{\frac{m}{\xi}} \sum_{k=0}^{N} b_k \xi^k d\xi . \tag{19}$$

The first set of terms can immediately be integrated; it gives

$$\tfrac{1}{2} \sum_{i=0}^{N} \frac{m^i}{i!} \sum_{k=0}^{N-i} \frac{1+(-1)^k}{k+1} b_{i+k} . \tag{20}$$

The second set contains the integrals

$$K(k)=\int_{-1}^{\infty} e^{\xi}\overset{m}{\xi^k}d\xi\,. \qquad (k=0,\,1,\,\ldots\,.\,N).$$

Making the transformation

$$u=-\frac{m}{\xi}\,; \qquad du=\frac{m}{\xi^2}d\xi\,;$$

we find

$$K(k)=(-1)^k m^{k+1}\int_m^{\infty}\frac{e^{-u}}{u^{k+2}}du\,.$$

These integrals again can be reduced on the logarithm-integral by the general formula

$$\int_a^b\frac{e^{-u}}{u^n}du=\sum_{i=1}^{n-1}\frac{(-1)^i(n-1-i)!}{(n-1)!}\left[\frac{e^{-u}}{u^{n-i}}\right]_a^b+\frac{(-1)^{n-1}}{(n-1)!}\int_a^b\frac{e^{-u}}{u}du\,.$$

Consequently

$$K(k)=\frac{1}{(k+1)!}\left\{e^{-m}\sum_{i=0}^{k}(-1)^{k-i}(k-i)!\,m^i-m^{k+1}\int_m^{\infty}\frac{e^{-u}}{u}du\right\}. \qquad (21)$$

Introducing (20) and (21) into (19) we obtain in conclusion

$$G(m)=\tfrac{1}{2}\sum_{i=0}^{N}\frac{m^i}{i!}\sum_{k=0}^{N-i}\frac{1+(-1)^k}{k+1}b_{i+k}-\frac{e^{-m}}{2}\sum_{i=0}^{N}m^i\sum_{k=0}^{N-i}\frac{(-1)^k k!}{(i+k+1)!}b_{i+k}$$
$$+\tfrac{1}{2}\int_m^{\infty}\frac{e^{-u}}{u}du\cdot\left\{\sum_{k=0}^{N}\frac{b_k m^{k+1}}{(k+1)!}\right\}. \qquad (22)$$

We have thus shown how to determine the emissivity function $H(m)$, the collustrivity function $G(m)$, and the intensity $J(m,\,\xi)$ of the radiation in each specified direction and at each specified layer. It remains to show that the solution obtained is unique.

Otherwise there would exist at least two different functions H and H^* both satisfying the equation (7) in the special case giving the radiation outside the solar atmosphere:

$$J(o, \xi) = \int_o^\infty \frac{e^{-\frac{m}{\xi}}}{\xi} H(m)dm = \int_o^\infty \frac{e^{-\frac{m}{\xi}}}{\xi} H^*(m)dm \,.$$
$$\xi > o$$

But, as we shall see, these equations are inconsistent, unless $H = H^*$.

On subtracting the second equation from the first, we find

$$o = \int_o^\infty \frac{e^{-\frac{m}{\xi}}}{\xi} (H - H^*)dm. \tag{23}$$

But according to the theorem of Weierstrass quoted above it is always possible to find two polynomials

$$P(m) = \sum_{i=o}^{N} a_i m^i$$

and

$$P^*(m) = \sum_{i=o}^{N} a_i^* m^i$$

such as to render each of the differences

$$|H(m) - P(m)|$$

and

$$|H^*(m) - P^*(m)|$$

less than $\epsilon/4$ for every m within a finite interval $o \leqq m \leqq M$, ϵ being any positive quantity given in advance. Hence

$$\left| \int_o^M \frac{e^{-\frac{m}{\xi}}}{\xi} (P - P^*)dm \right| + \int_M^\infty \frac{e^{-\frac{m}{\xi}}}{\xi} (H - H^*)dm < \frac{\epsilon}{2} \,.$$

Moreover, by (23)

$$\left| \int_M^\infty \frac{e^{-\frac{m}{\xi}}}{\xi} (H - H^*)dm \right| < \frac{\epsilon}{4},$$

provided M is taken sufficiently great. Consequently

$$\left| \int_0^M \frac{e^{-\frac{m}{\xi}}}{\xi} (P-P^*)dm \right| < \frac{3\epsilon}{4};$$

or on integration

$$\left| \sum_{i=0}^{N} i!\,(a_i-a_i^*)\xi^i \left\{ 1-e^{-\frac{M}{\xi}} \sum_{k=0}^{i} \frac{1}{k!}\left(\frac{M}{\xi}\right)^k \right\} \right| < \frac{3\epsilon}{4}.$$

Now in virtue of a theorem due to Abel the module of the series

$$\sum_{i=0}^{N} a_i u_i$$

never exceeds the quantity

$$a_0 \left| \sum_{i=0}^{N} u_i \right|,$$

provided the quantities a_i are positive and decreasing. The left-hand side of the inequality above is therefore less than

$$(1-e^{-\frac{M}{\xi}}) \left| \sum_{i=0}^{N} i!\,(a_i-a_i^*)\xi^i \right|;$$

but M is a large number and therefore $e^{-\frac{M}{\xi}}$ is certainly less than $\frac{1}{4}$; and in conclusion

$$\left| \sum_{i=0}^{N} i!\,(a_i-a_i^*)\xi^i \right| < \epsilon. \qquad (24)$$

A well-known theorem now states that if a power series is identically zero within a finite interval, each separate coefficient must be zero; hence:

$$\lim_{\epsilon=0} |\, a_i-a_i^* \,| = 0 \qquad \therefore \quad \lim_{N=\infty} P^*(m) = \lim_{N=\infty} P(m)$$

or finally

$$H^*(m) = H(m)$$

which establishes the result stated above.

The function $H(m)$ and in consequence the functions $G(m)$ and $J(m, \xi)$ can thus be determined without ambiguity by the aid of the method explained in this paragraph. Before pursuing the discussion of these functions in their bearings on the optics of the photosphere we are, however, obliged to work out the computations numerically.

3. NUMERICAL COMPUTATION OF THE EMISSIVITY AND COLLUSTRIVITY FUNCTIONS AND THE INTENSITY OF THE RADIATION

The most complete set of observations concerning the intensity distribution along the diameter of the sun's disk is furnished by the careful and laborious investigations of Abbot, Fowle, and Aldrich[1] of the Smithsonian Institution at Mount Wilson. Their researches will be the base of the following computations.

From the point of view of this paper the principal desiderata as to the observations are the following:

1. The accuracy must be sufficiently high to enable us to evaluate a satisfactory number of coefficients in the expansion (14). The observations quoted admit of fixing the four first coefficients b_0 to b_3.

2. The observations must be continued to the immediate vicinity of the sun's limb. The Smithsonian material only extends to 0.95 of the radius of the disk, corresponding to the interval $1 \geqq \xi \geqq 0.3123$; this fact necessitates some disadvantageous extrapolations.

3. The material must be homogeneous. There are indications of a slight variability of the intensity distribution from day to day and from year to year. Therefore each day of observation ought to be represented in the mean for each wave-length and not only in a few of the means. This condition is not fulfilled, the number of

[1] C. G. Abbot, F. E. Fowle, and L. B. Aldrich, *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **3**, 157, 1913.

observations being, e.g., for $\lambda = 0''.386$ only three, but for $\lambda = 0''.501$ more than a hundred.

4. The wave-length to be observed must be selected in a suitable manner so that the Fraunhofer lines intervene as little as possible: for even if the disturbances of the intensity distribution by the lines perhaps are negligible, owing to the remarkable constancy of the contrast between the lines and their spectral surroundings all over the sun's disk, the absolute amount of the radiation is naturally affected by them.

In a later publication of the Smithsonian Institution[1] there are to be found some results of more recent investigations; these seem to be more adequate to our requirements. Although the main features of our results most likely should be unaltered, I regret that I cannot find time to renew the computations utilizing these observations, since they illustrate another part of the sun-spot period.

As stated above, the calculations are to be commenced by finding the polynomial approximations. For this purpose I have used the method of developing intermediately in series of Legendrian polynomials.[2] The merit of this method lies in the fact that every following approximation is obtained simply by adding one term without altering the coefficients of the preceding ones.

The coefficients b obtained in this manner are included in Table I. They are determined so as to suit the graphical representation of the function $J(0, \xi)$ as well as possible within the interval $0.3 \leqq \xi \leqq 1$. (Evidently we have $\xi = \sqrt{1-\rho^2}$, where ρ is the distance of an element from the center of the disk as a fraction of the radius.)

The first line in the table refers to observations made by Schwarzschild and Villiger.[3] The units are the same as used in the original papers, the intensity at the center being equated to one for each wave-length apart. The remaining differences between the observed data and the polynomial (14) hardly present any systematic features.

[1] C. G. Abbot, F. E. Fowle, and L. B. Aldrich, *Smithsonian Miscellaneous Collections*, **66**, No. 5, 1916.

[2] H. v. Sanden, *Praktische Analysis*, p. 113, Leipzig, 1914.

[3] K. Schwarzschild und W. Villiger, *Astrophysical Journal*, **23**, 284, 1906.

From the data given in Table I the emissivity function $H(m)$ and the collustrivity function $G(m)$ can immediately be found in virtue of the relations (16) and (22). The results are collected in

TABLE I

THE COEFFICIENTS OF THE POLYNOMIAL $J(o, \xi)$

λ	b_0	b_1	b_2	b_3
$0^{\mu}.323$	+0.1269	+0.7625	+0.2109	−0.1283
0.386	+0.1699	+0.7553	+0.1244	−0.0525
0.433	+0.2126	+0.7002	+0.1903	−0.1108
0.456	+0.1786	+0.9939	−0.2026	+0.0292
0.481	+0.2192	+0.9638	−0.2200	+0.0350
0.501	+0.2444	+0.9507	−0.2610	+0.0641
0.534	+0.3119	+0.7694	−0.0312	−0.0525
0.604	+0.3288	+0.9794	−0.4819	+0.1749
0.670	+0.3566	+1.0359	−0.6133	+0.2216
0.699	+0.3716	+1.0215	−0.6146	+0.2216
0.866	+0.4563	+0.9499	−0.6450	+0.2391
1.031	+0.5163	+0.8220	−0.4984	+0.1633
1.225	+0.5572	+0.7756	−0.4972	+0.1633
1.655	+0.6491	+0.6726	−0.5019	+0.1808
2.097	+0.6851	+0.6322	−0.5139	+0.1983

the subsequent tables. The second line of the head of the tables contains the corresponding values of a new variable p defined by the relation

$$p = \frac{m}{m+1};$$

this variable is preferable to m for graphical representation, because it runs through the finite interval o to 1 at the same time that m runs from o to ∞. The last column $m=4$ is naturally not very exact; only the e^{-4}th part, i.e., 0.018, of a beam originating in that layer and going straight outward reaches the face.

Finally the radiation at various layers and in various directions can be calculated from (17) and (18). To give a survey I have indicated the results graphically in the figures below. The intensity in a specified direction is proportional to the radius vector in that same direction. The inmost curve of each figure illustrates the radiation at the external limit of the absorbing layers ($m=0$); then there follow consecutively the curves $m=0.429$, $m=1$, and

$m = 2.333$. Of course it must be remembered that m is a function of the wave-length. In section 5 we shall examine the relation between the optical masses with respect to different wave-lengths.

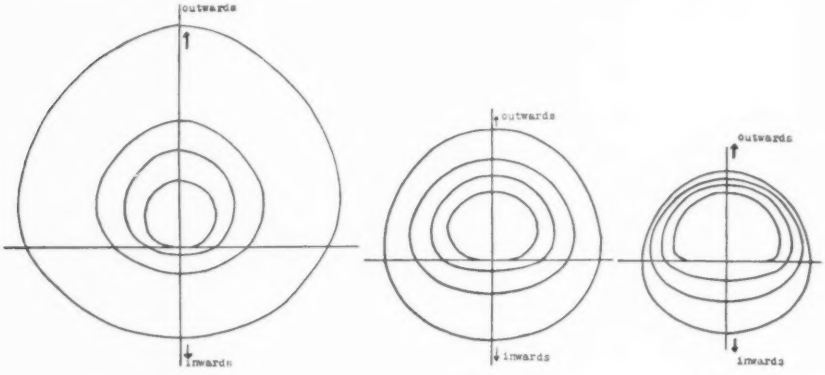

FIG. 1 $\lambda = 0^{\mu}.386$

FIG. 2 $\lambda = 0^{\mu}.670$

FIG. 3 $\lambda = 1^{\mu}.655$

The intensity as a function of the direction of the beams

4. SCATTERING, OR REAL ABSORPTION?

As mentioned in the introduction there are very different opinions about the importance of scattering in solar optics. By the aid of the results obtained in the preceding paragraph we are now going to examine the various possibilities.

We begin with the extreme assumption that there is a vast atmosphere, scattering, but not absorbing, surrounding the seat of emission in such powerful layers as to cut off all direct radiation from within practically completely.

The coefficient of real absorption then must be zero throughout the atmosphere, and accordingly, by (5)

$$H(m) = G(m). \tag{25}$$

Moreover, by (2), (7), and (8):

$$G(m) = \tfrac{1}{2}\int_{0}^{+1} e^{\frac{m}{\xi}} d\xi \int_{m}^{\infty} \frac{H(m)}{\xi} e^{-\frac{m}{\xi}} dm - \tfrac{1}{2}\int_{-1}^{0} e^{\frac{m}{\xi}} d\xi \int_{0}^{m} \frac{H(m)}{\xi} e^{-\frac{m}{\xi}} dm\,;$$

or applying a simple transformation due to King and Schwarzschild:

$$G(m)=\tfrac{1}{2}\int_0^\infty H(\mu)d\mu\int_{|m-\mu|}^\infty \frac{e^{-u}}{u}du\,;$$

or finally by (25)

$$G(m)=\tfrac{1}{2}\int_0^\infty G(\mu)d\mu\int_{|m-\mu|}^\infty \frac{e^{-u}}{u}du\,; \qquad (26)$$

where μ and u are integration variables. Since the equation (26) is an integral equation of Fredholm's second type, it admits of a unique solution; and since the wave-length does not occur in (26), this solution must be the same function of the optical mass for all

TABLE II

THE EMISSIVITY FUNCTION, $H(m)$

λ	$m=0$	0.111	0.25	0.429	0.667	1	1.5	2.333	4
	$p=0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0$^{\mu}$.323...	0.127	0.213	0.324	0.471	0.676	0.974	1.436	2.209	3.495
0.386...	0.170	0.255	0.363	0.504	0.698	0.979	1.413	2.165	3.623
0.433...	0.213	0.292	0.393	0.529	0.716	0.990	1.415	2.129	3.352
0.450...	0.179	0.288	0.421	0.586	0.798	1.076	1.458	2.008	2.847
0.481...	0.219	0.325	0.453	0.613	0.815	1.079	1.437	1.943	2.686
0.501...	0.244	0.348	0.474	0.629	0.823	1.076	1.413	1.888	2.644
0.534...	0.312	0.397	0.503	0.638	0.815	1.057	1.401	1.910	2.577
0.604...	0.329	0.435	0.559	0.707	0.883	1.096	1.354	1.673	2.259
0.670...	0.357	0.468	0.597	0.747	0.922	1.123	1.345	1.573	1.955
0.699...	0.372	0.481	0.608	0.756	0.927	1.123	1.337	1.551	1.902
0.866...	0.456	0.558	0.674	0.807	0.958	1.124	1.290	1.424	1.650
1.031...	0.516	0.605	0.707	0.825	0.962	1.116	1.280	1.423	1.558
1.225...	0.557	0.640	0.736	0.846	0.972	1.111	1.253	1.359	1.423
1.655...	0.649	0.721	0.802	0.894	0.995	1.101	1.195	1.234	1.250
2.097...	0.685	0.752	0.828	0.911	1.002	1.093	1.167	1.182	1.220

wave-lengths. Hence, summing up the contents of (25) and (26): *If the extinction is due to pure scattering all the way through the atmosphere, the emissivity and collustrivity functions must be equal and independent of the wave-length.*

Tables II and III demonstrate that this is not the case. The assumption is thereby proved to be erroneous.

Next, suppose that the interior layers are absorbing as well as scattering, but the external layers (say $0<m<m^*$) exclusively scattering. Then the equation (25) obviously subsists within the same interval $0<m<m^*$; but (26) naturally not. Hence *if the sun is enveloped by an exclusively scattering layer, the emissivity and collustrivity functions must coincide within the corresponding interval of m.*

If the scattering layer were equal to the earth's atmosphere as to its optical properties, the interval in question would be around 0.15 in the visible spectrum and 1.25 in the extreme ultra-violet.

TABLE III

THE COLLUSTRIVITY FUNCTION $G(m)$

λ	$m=0$	0.111	0.25	0.429	0.667	1	1.5	2.333	4
	$p=0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0$^{\mu}$323...	0.273	0.346	0.438	0.566	0.748	1.020	1.452	2.183	3.396
0.386...	0.288	0.365	0.459	0.585	0.762	1.024	1.440	2.165	3.598
0.433...	0.299	0.380	0.474	0.598	0.771	1.027	1.428	2.109	3.270
0.456...	0.308	0.393	0.497	0.633	0.816	1.069	1.432	1.973	2.818
0.481...	0.318	0.408	0.512	0.645	0.822	1.064	1.406	1.906	2.662
0.501...	0.324	0.416	0.520	0.651	0.824	1.057	1.383	1.860	2.643
0.534...	0.337	0.431	0.532	0.657	0.821	1.048	1.375	1.864	2.500
0.604...	0.351	0.452	0.559	0.688	0.847	1.050	1.311	1.659	2.331
0.670...	0.363	0.469	0.578	0.708	0.865	1.057	1.286	1.554	2.050
0.699...	0.367	0.474	0.583	0.712	0.867	1.055	1.277	1.531	1.997
0.866...	0.388	0.502	0.612	0.735	0.878	1.043	1.224	1.405	1.752
1.031...	0.401	0.519	0.627	0.745	0.881	1.039	1.215	1.392	1.612
1.225...	0.410	0.531	0.638	0.753	0.883	1.028	1.185	1.327	1.477
1.655...	0.432	0.560	0.666	0.774	0.889	1.011	1.129	1.213	1.329
2.097...	0.440	0.571	0.676	0.781	0.890	1.001	1.104	1.169	1.315

Once more referring to Tables II and III we find a marked discrepancy between the stated conclusion and the experimental data, especially in the extreme ultra-violet region. The underlying assumption must therefore be false again.

We now proceed to apply our results to the hypothesis of pure absorption, assuming that the whole of the dissipating energy is converted into heat and that the radiation is a pure temperature radiation. On this hypothesis we have by (5)

$$H(m)=E(m); \tag{27}$$

i.e., *if the scattering is negligible, the emissivity function is equal to the emissive power of an absolutely black body of the temperature of the layer.*

In virtue of Planck's law the emissive power of an absolutely black body is a known function of the temperature; we have in a frequent notation

$$E=\frac{2c^2h}{\lambda^5}\cdot\frac{1}{e^{\frac{ch}{\kappa T\lambda}}-1},$$

or, using the C.G.S. units:

$$E=\frac{1.177\cdot 10^{-5}}{\lambda^5\left(e^{\frac{1.43}{\lambda T}}-1\right)}. \tag{28}$$

By means of this equation the absolute temperature of each layer can be computed as a function of the overlying optical mass.

If our assumption concerning the nature of the extinction is true, the temperature of the outermost layer $m=0$ must be one and the same, irrespective of the wave-length used in the computations. We shall utilize this circumstance to test the assumption.

To begin with, we have to reduce $H(0)$ to C.G.S. units. The units employed above were defined as the radiation issued within the unit solid angle from a body which radiates like the central parts of the sun's disk. Now the energy-curve of the solar radiation outside the earth's atmosphere is determined at Mount Wilson[1] in some arbitrary unit; on interpolation we have obtained the figures given in the second column of Table IV. As the solar constant is accurately measured (1.932 cal. per cm² and min. $=1.348\cdot 10^6$ erg per cm² and sec), we can readily put them into absolute units. The numbers thus obtained would give the absolute intensity of the radiation from the entire solar disk at the wave-lengths specified in the table. Since the solid angle occupied by the sun is $6.796\cdot 10^{-5}$ (assuming the sun's semidiameter to be 959".7), the radiation from a body which emits like the sun but occupies the unit solid angle is found by dividing by that quantity. In order to account for the fact that the measurements bear upon the joint radiation of the

[1] *Annals of the Smithsonian Astrophysical Observatory*, **3**, 197, 1913.

entire solar disk but the units of the preceding paragraph upon the radiation at the center, we have further to divide by the quantity

$$\frac{2\pi \int_0^1 J \cdot \rho d\rho}{\pi \cdot 1^2} = 2 \int_0^1 J(0, \xi) \cdot \xi d\xi = 2 \sum_{i=0}^{N} \frac{b_i}{i+2};$$

$\rho = \sqrt{1-\xi^2}$ denoting the distance of a zonal element from the center of the disk. By this procedure the units will be expressed in the C.G.S. system; they are given in the third column of Table IV and are there denoted U_λ. Finally, multiplying the value $H(0)$ obtained above with the corresponding U_λ, we get $H(0)$ expressed in ergs per cm² and sec, as tabulated in the fourth column of Table IV.

TABLE IV

REDUCTION OF $H(0)$ TO C.G.S. UNITS

λ	C	$U_\lambda \cdot 10^{-14}$	$H(0) \cdot 10^{-13}$ (abs.)	λ	C	$U_\lambda \cdot 10^{-14}$	$H(0) \cdot 10^{-13}$ (abs.)
0.323....	1180	1.014	1.29	0.670....	4060	2.887	10.29
0.386....	3590	2.962	5.03	0.699....	3660	2.589	9.62
0.433....	5490	4.437	9.43	0.866....	2270	1.556	7.10
0.456....	6140	4.824	8.62	1.031....	1540	1.011	5.34
0.481....	6200	4.780	10.48	1.225....	1030	0.686	3.82
0.501....	6060	4.624	11.30	1.655....	480	0.311	2.02
0.534....	5780	4.326	13.50	2.097....	190	0.124	0.85
0.604....	4990	3.632	11.94				

The temperature of the layer $m = 0$ is now readily found by (28). The results are given in the tabular summary below. The departures from the mean cannot be said to be small, but I think they might be entirely accounted for by the inaccuracy of the experimental material. The precision of the observations is not sufficiently great to enable us to evaluate higher terms than the fourth in the expansion of $J(0, \xi)$. Therefore, if the outermost layers of the sun only give rise to a short interval of m with feeble radiation, they are not revealed by the computations; so it is to be expected that the apparent limiting temperatures will come out higher, the lower the absorption coefficient is, and vice versa. We shall now determine the values of the absorption coefficients in the sequel;

utilizing these values in anticipation we give below the graphical run of the absorption coefficients of the outermost layer together with the difference between the mean and the separate values of Table V.

TABLE V

THE ABSOLUTE TEMPERATURE OF THE LAYER $m=0$

λ	T	λ	T
0$^{\mu}$323.....	4360°	0$^{\mu}$670.....	4800°
0.386.....	4080	0.699.....	4750
0.433.....	4920	0.866.....	4640
0.456.....	4790	1.031.....	4640
0.481.....	4890	1.225.....	4670
0.501.....	4920	1.655.....	4960
0.534.....	5040	2.097.....	4580
0.604.....	4910		

Mean.....4770° abs.

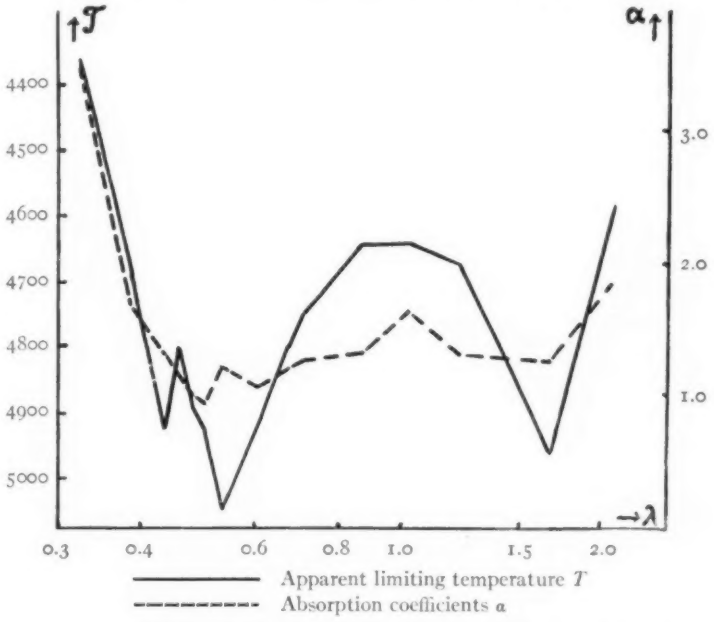

FIG. 4.—Apparent limiting temperature and absorption coefficients

The real temperature of the utmost layer which can make itself perceptible by its absorption and radiation is most correctly indicated by the lowest number of the table. Because of the Fraun-

hofer lines that number should be raised a little.[1] The most likely value of the limiting temperature therefore appears to be around 4500° abs.

It is evident that no real objection against the postulated conception of the extinction can be made from the discussion above. But there remains yet an assumption to be examined, namely, the one that both the coefficients of absorption and scattering are different from zero.

This being the case, we should have by (5)

$$E=H\left(1-\frac{\beta}{a}\frac{G-H}{H}\right). \tag{29}$$

The scattering must not render the expression within the parenthesis negative, which enables us to find a superior limit of β:

$$\beta\leqq\frac{aH}{G-H}$$

or, in particular, for the wave-length $0^{\mu}323$:

$$\beta\leqq 0.867\,a\,. \tag{30}$$

Having thus found a superior limit for the shortest wave-length, we can readily assign a superior limit for each of the longer wave-lengths too. According to Lord Rayleigh's law

$$\beta=\frac{32}{3}\pi^3\frac{(n-1)^2}{N\lambda^4}, \tag{31}$$

the selective properties of scattering are in the main due to the factor λ^4 of the denominator. It may be that the formula is not exactly valid; the factor λ^4 is safe in any case, since it can be deduced from pure dimensional considerations.[1] The quantity

$$\beta_\lambda=\beta_{0.323}\cdot\frac{0.323^4}{\lambda^4} \tag{32}$$

[1] According to Rowlands' spectral atlas, the absorption by the lines in the region about $0^{\mu}323$ does not seem to be very much more extensive than in the visible region.

[1] Lord Rayleigh (J. W. Strutt), *Philosophical Magazine* (5), **41**, 110, 1871.

is therefore a superior limit of the scattering coefficient of the wavelength λ. If the whole of (31) were exact, the limit would be still smaller.

Thus, the extinction in the sun can be caused either by pure absorption, or by absorption together with scattering within the indicated limits. In any case the scattering is negligible within the red and infra-red parts; the question whether it is also negligible within the remaining parts of the visible spectrum and the ultra-violet must be left unsettled if we wish to avoid entering into somewhat hypothetical considerations as to the state and optical behavior of the layers.

To get, nevertheless, at least an idea of the probable value of the scattering coefficients, notice in the first place that the limit above is very wide, so wide in fact that it even admits the temperature of the outmost layer to be zero on the absolute scale; I need not point out that this temperature and any low temperature is absurd in the overwhelming ardor of the radiation from within;[1] in particular, venturing the assumption that the layer is in radiative equilibrium, we should be able to reduce the maximum of the scattering to a small fraction of the foregoing amount; most likely the scattering is negligible all through the spectrum.

In one of his celebrated memoirs on solar physics Schwarzschild[2] appears to be disposed in favor of the scattering theory of the solar atmosphere; his opinion is based on a study of the Fraunhofer lines. There is clearly no contradiction between his result and mine; for the layers with which we have been concerned above are those giving rise to the continuous spectrum, i.e., the photospheric layers, while Schwarzschild's result bears upon the optics of the reversing layer with its discontinuous absorption and radiation; according to Julius' measurements during the annular eclipse of 1912 the total amount of radiation from the entire solar atmosphere outside the photosphere is 0.001 at most of the sun's total emission; such small amounts are not discovered by our computations.

[1] From a spectroscopic determination the temperature of the reversing layer is found to be 4300° abs. R. T. Birge, *Astrophysical Journal*, **54**, 273, 1922.

[2] K. Schwarzschild, *op. cit.*, p. 1200, 1914.

5. THE TEMPERATURE OF THE PHOTOSPHERIC LAYERS

The temperature of any layer can be computed from the equation

$$E=H\left(1-\frac{\beta}{a}\frac{G-H}{H}\right), \tag{29}$$

provided either β/a is known or $\frac{G-H}{H}$ is a small quantity. From Tables II and III we find that the latter is the case for the deep layers, at least as a good approximation, so we have simply

$$E=H. \tag{27}$$

Inversely, having computed E for a specified temperature by means of Planck's formula, we know by (27) the function $H(m)$ for the layer where that temperature exists, and can then read off

TABLE VI

OPTICAL MASS AND TEMPERATURE

λ	T						
	5100°	5500°	5900°	6300°	6700°	7100°	7500°
0$^{\mu}$.323	0.53	1.09	1.90	3.20	.....	.....	.....
0.386	0.21	0.48	0.87	1.36	1.97	2.69	3.54
0.433	0.09	0.30	0.59	0.92	1.32	1.79	2.36
0.456	0.08	0.25	0.46	0.74	1.11	1.59	2.19
0.481	0.07	0.23	0.44	0.72	1.09	1.56	2.16
0.501	0.08	0.23	0.45	0.74	1.11	1.59	2.19
0.534	0.03	0.22	0.48	0.78	1.17	1.61	2.15
0.604	0.07	0.24	0.47	0.78	1.22	1.66	2.69
0.670	0.11	0.30	0.54	0.87	1.38	2.36	3.74
0.699	0.13	0.33	0.59	0.95	1.55	2.66	.....
0.866	0.22	0.43	0.76	1.24	2.45	.....	.....
1.031	0.23	0.49	0.80	1.27	2.28	.....	.....
1.225	0.22	0.43	0.72	1.13	1.99	.....	.....
1.655	0.05	0.25	0.49	0.80	1.26	.....	.....
2.097	0.27	0.54	0.93	.....	.....	.....	.....

the corresponding values of m, on a set of diagrams representing Table II. In this way Table VI has been deduced. From this table we can obtain the correspondence between the optical masses with regard to different wave-lengths.

By the aid of the table we can further trace a kind of isothermals, which are very instructive and make the optical conditions easy to

survey. In Figure 5 λ and m are co-ordinates, and the curves connect points with equal temperature. The scale of wave-lengths is a logarithmic one. From the figure it is obvious that an appreciable

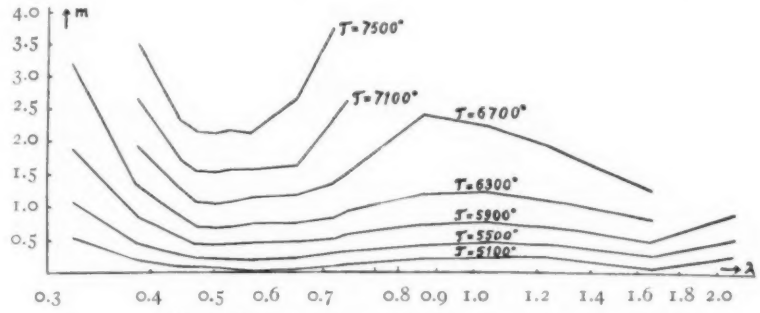

FIG. 5.—Isothermals: $T=5100$, 5500, 5900, 6300, 6700, 7100, and 7500

part of the green and the blue light comes from deep and hot layers, while the radiation of the ultra-violet and part of the infra-red spectrum in the main is due to cooler external layers.

6. THE EXTINCTION COEFFICIENTS

The optical mass was defined above by the differential relation

$$dm_\lambda = -(a_\lambda + \beta_\lambda)dr = -\gamma_\lambda dr\,, \qquad (3)$$

where the coefficient γ—the sum of the coefficients of absorption and scattering—might be called the coefficient of extinction. The first differences of Table VI measure by (3) the integral

$$m(T_2) - m(T_1) = \int_{T_2}^{T_1} \gamma dr\,,$$

taken between the layers indicated by their temperatures T_1 and T_2.

Let l_{12} be the distance between the layers T_1 and T_2. We have then approximately

$$\int_{T_2}^{T_1} \gamma dr = l_{12} \cdot \gamma \left\{ \frac{T_1+T_2}{2} \right\},$$

or

$$m(T_2)-m(T_1)=l_{12}\cdot\gamma\left\{\frac{T_1+T_2}{2}\right\},$$

where $\gamma\left\{\frac{T_1+T_2}{2}\right\}$ denotes the extinction coefficient at the layer $\frac{T_1+T_2}{2}$. From this equation γ may be computed for each of the layers $T=5300$, 5700, 6100, 6500, 6900, and 7300°; however, as we do not know l_{12}, only in relative numbers. Table VII contains the ratio of the extinction coefficient of the wave-length λ to that of the wave-length $0^{\mu}.481$.

TABLE VII

EXTINCTION-COEFFICIENTS

λ	T					
	5300°	5700°	6100°	6500°	6900°	7300°
$0^{\mu}.323$	3.50	3.86	4.64	.....	.....	.....
0.386	1.69	1.86	1.75	1.65	1.53	1.42
0.433	1.31	1.38	1.18	1.08	1.00	0.95
0.456	1.06	1.00	1.00	1.00	1.02	1.00
0.481	1.00	1.00	1.00	1.00	1.00	1.00
0.501	0.94	1.05	1.04	1.00	1.02	1.00
0.534	1.19	1.24	1.07	1.05	0.94	0.90
0.604	1.06	1.10	1.11	1.19	0.94	1.72
0.670	1.19	1.14	1.18	1.38	2.09	2.30
0.699	1.25	1.24	1.29	1.62	2.36	.....
0.866	1.31	1.57	1.71	3.27	.....	.....
1.031	1.63	1.48	1.68	2.73	.....	.....
1.225	1.31	1.38	1.46	2.32	.....	.....
1.655	1.25	1.14	1.11	1.24	.....	.....
2.097	1.86	.....	.....	.....	.....	.....

If the extinction is entirely due to real absorption, the data of Table VII naturally denote absorption coefficients. Otherwise, they are maximal values of the absorption coefficients. Assuming that the quotients of the extinction coefficients of the outmost layer are equal to those of the layer $T=5300°$ we can also give minimal values for that layer and at the same time maximal values of its scattering coefficients. From (30) and (32) we obtain the results contained in Table VIII. The extinction coefficient of $\lambda=0^{\mu}.481$ is still retained as unit. I repeat that the actual values of β/α must

be much lower than those of the table; for if these were correct, we should obtain quite incongruous values of the limiting temperature.

TABLE VIII

MINIMUM OF ABSORPTION AND MAXIMUM OF SCATTERING

λ	α min.	β max.	λ	α min.	β max.
0$^{\mu}$.323	1.87	1.63	0$^{\mu}$.670	1.10	0.09
0.386	0.89	0.80	0.699	1.18	0.07
0.433	0.81	0.50	0.866	1.28	0.03
0.456	0.65	0.41	1.031	1.62	0.01
0.481	0.67	0.33	1.225	1.30	0.01
0.501	0.66	0.28	1.655	1.25	0.00
0.534	0.97	0.22	2.097	1.86	0.00
0.604	0.93	0.13			

7. THE HEAT ECONOMY OF THE PHOTOSPHERIC LAYERS

Several of the papers quoted at the outset start with the assumption that the external layers of the photosphere are in radiative equilibrium, i.e., the energy gain by absorption is equivalent to the energy loss by emission:

$$\int_0^\infty \alpha G d\lambda = \int_0^\infty \alpha E d\lambda .$$

We are now able to test this assumption. Provided the extinction is a real absorption all through, we have $\alpha = \gamma$, so we can compute the integrals from Tables III and VII. The ratio

$$q = \left\{ \int_0^\infty \alpha (G - E) d\lambda \right\} : \left\{ \int_0^\infty \alpha E d\lambda \right\}$$

is evidently an index of the departures from the radiative equilibrium. By graphical integration I have found the following values:

T	q
5300°	−0.019
5700°	−0.025
6100°	−0.037
6500°	−0.034

The differences are most likely entirely caused by inevitable inaccuracies; and so we obtain, as our last conclusion, the verification of the very starting-point of most of the previous investigations.

LULEÅ, SWEDEN
September 1922

Minor Contributions and Notes

NOTES ON TWO STARS HAVING VARIABLE BRIGHT LINES

ABSTRACT

Two spectroscopic binaries of class B, 47 ω Orionis and 43 θ² Orionis, previously known to have bright lines at Hβ, were observed at the Yerkes Observatory in 1922 and 1923. The *intensity* of the bright lines is found to vary in the course of a few days.

47 ω Orionis ($\alpha = 5^h34^m$, $\delta = 4°4'$, 1900; mag., 4.5; spectral type, B3) has exceedingly wide and diffuse absorption lines, except the K line, which is sharp and narrow. Hγ is fairly good for measurement. This star was found to be a spectroscopic binary by Professor E. B. Frost.[1]

Paul W. Merrill[2] noticed in 1912 on two plates, taken on September 23 and October 14, that Hα was a strong bright line, while Hβ was a well-defined double, bright superposed on absorption.

In a paper presented at the twenty-second meeting of the American Astronomical Society in 1919, C. D. Perrine[3] classifies ω Orionis among the stars that have lost bright Hβ. His data are based on objective-prism photographs taken with the astrographic refractor and a 20° prism (in Córdoba).

In 1921 the star was observed by F. Henroteau[4] at Ottawa. Two plates taken on January 25 show a rather weak emission line, which is certainly single, in the middle of the absorption line at Hβ.

In the *Henry Draper Catalogue* the following remark refers to ω Orionis (No. 37490): "No bright lines are seen on a photograph taken with the 11-inch telescope on January 31, 1906."

Forty-five spectrograms of ω Orionis were obtained with the Bruce Spectrograph of the Yerkes Observatory between October 30, 1922, and March 31, 1923. The first plate on October 30 showed a

[1] *Lick Observatory Bulletin,* **181,** 22, 1910.

[2] *Ibid.,* **237,** 168, 1913.

[3] *Popular Astronomy,* **27,** 90, 1919.

[4] *Publications Dominion Observatory,* Ottawa, **5,** 337.

single bright line at Hβ, while the next plate on November 6 had two bright components of unequal intensity. The later plates confirmed the changes of the intensity of the two components, which occur in rather short intervals of a few days. Some of the plates entirely fail to show the emission. Preliminary measures indicate that the changes in the emission lines are not due to orbital motion of the binary system, for the separation of the two components apparently does not vary, averaging about 240 km/sec. It seems probable that the two components really vary in intensity. Hγ does not show any trace of emission on our plates, but the velocity derived from it differs by about 30–40 km/sec, from that of the K line and *He* 4472 and agrees very well with the velocity determined from the mean of the two bright components of Hβ. This seems to indicate that Hγ is also affected by emission which does not stand out on the background of the continuous spectrum.

The presence of bright Hβ had previously been found by Professor Frost from some of the early plates, numbering eleven and taken between 1903 and 1912. But on none of these plates does the bright component appear so strong as on most of the plates of 1922–1923. Per contra, most of the early plates show a wide absorption line at Hβ without any trace of emission, while the great majority of plates taken this season show two bright components. This indicates that in addition to the rapid changes in the spectrum there has been a slow change in the appearance of Hβ during the past twenty years. Observations of this star will be resumed in the autumn.

43 θ^2 Orionis ($\alpha = 5^h30^m$, $\delta = -5^\circ 29'$, 1900; mag., 4.9; spectral type, B1). The lines of this star are rather poor for measurement, except K, which is sharp. It was announced as a spectroscopic binary by Frost and Adams[1] in 1904. No bright lines were visible on the plates taken at that time.

In 1919 F. Henroteau[2] found that the broad hydrogen absorption lines were divided by sharp and well-defined emission lines.

In February, 1922, three plates of this star were taken by Professor S. B. Barrett with the Bruce Spectrograph. No emission visible.

[1] *Astrophysical Journal*, **19**, 153, 1904.

[2] *Publications of the Dominion Observatory*, Ottawa, **5**, 19.

In 1923 six plates were taken by Mr. Barrett and myself. Some of them show a fairly strong, single emission line at Hβ, while others, though of good quality, fail to show any bright lines. No emission is visible on any of the other absorption lines. The variation in intensity of bright Hβ and the absence of emission at Hγ and of the nebular lines prove that the observed emission line is not due to the Orion Nebula.

It should be remembered that 43 θ^2 Orionis is not one of the trapezium stars. It is Bond 685 and follows the trapezium about 6^s, south 100″.

The star will be observed again next season.

OTTO STRUVE

YERKES OBSERVATORY
April 14, 1923